Insights on Loosely Bound Molecules from Atomic Physics

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ExoHad collaboration

Loosely Bound Molecule

2 constituents with reduced mass *m*

short-range interactions

S-wave scattering length *a*

loosely bound molecule

S-wave bound state with binding energy $|\varepsilon|$ (and width Γ)

small compared to energy scale set by the range:

$$|\varepsilon - i\,\Gamma/2| \ll 1/[m\,(\text{range})^2]$$

large scattering length *a*: $a \gg \text{range}$

universal properties determined by ε (or *a*)

- energy: $\varepsilon = -\,1/(m\,a^2)$
- wavefunction: $\psi(r) = (1/r)\,\exp(-r/a)$

most of the probability comes from *r* beyond range

Particles with Large Scattering Length

large S-wave scattering length *a*: $|a| \gg \text{range}$

a > *0*: bound state (loosely bound molecule)
a < *0*: virtual state

low-energy scattering amplitude: $f_k = \dfrac{1}{-1/a - i\,k}$

differential cross section: $\dfrac{d\sigma}{d\Omega} = \dfrac{1}{1/a^2 + k^2}$

Large low-energy cross section: $\sigma = 4\pi\, a^2 >> (\text{range})^2$

Particles with Large Scattering Length

large S-wave scattering length *a*: $|a| \gg \text{range}$

low-energy physics determined by *a*

well-behaved unitary limit $a \longrightarrow \infty$

$\Longrightarrow$ scale-invariant low-energy physics

differential cross section: $$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}$$

Effective Field Theory

nonrelativistic conformal field theory

with marginal operator that controls scattering length

(More) Fundamental Theory

unitary limit $a \longrightarrow \infty$

can be reached by tuning parameters: mass ?

interaction strength ?

Loosely Bound Charm-Meson Molecules

two exquisite examples!

$\chi_{c1}(3872)$ near $D^{*0}\bar{D}^0$ threshold

Breit-Wigner energy: $E_{\mathrm{BW}} = -0.07 \pm 0.12\ \mathrm{MeV}$

width: $\Gamma_{\mathrm{BW}} = 1.2 \pm 0.2\ \mathrm{MeV}$

$T_{cc}^{+}(3875)$ near $D^{*+}D^0$ threshold

Breit-Wigner energy: $E_{\mathrm{BW}} = -0.27 \pm 0.06\ \mathrm{MeV}$

width: $\Gamma_{\mathrm{BW}} = 0.4 \pm 0.2\ \mathrm{MeV}$

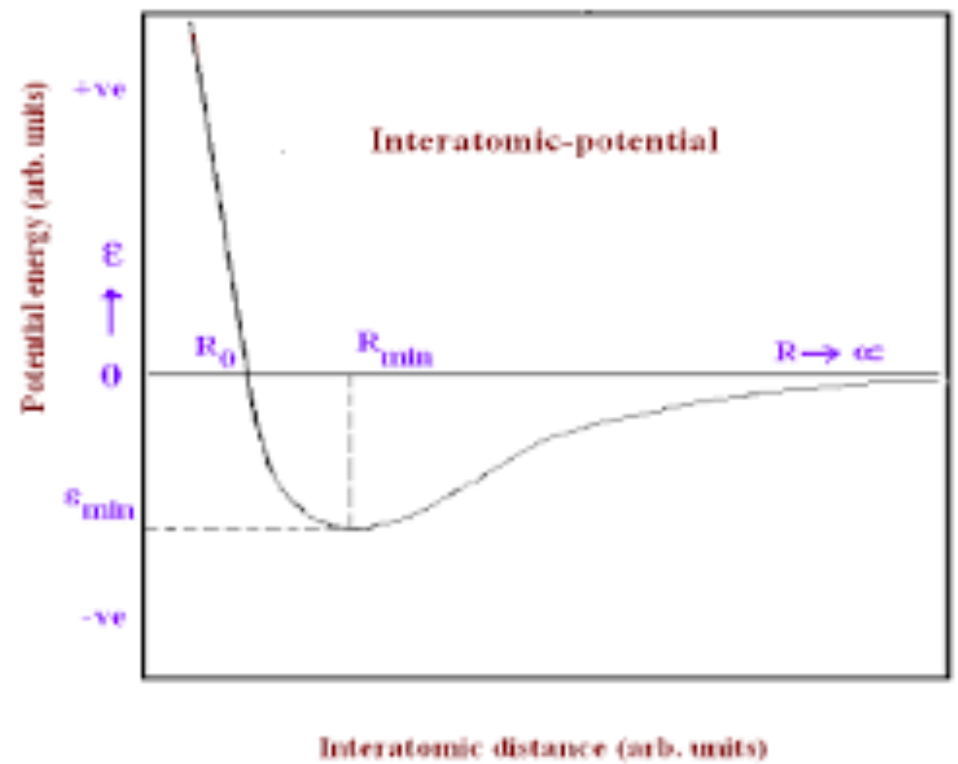
binding energy and width small compared to
energy scale from pion exchange: $m_\pi^2/m_D \approx 10\ \mathrm{MeV}$

Tuning parameters for unitary limit: $a \longrightarrow \infty$

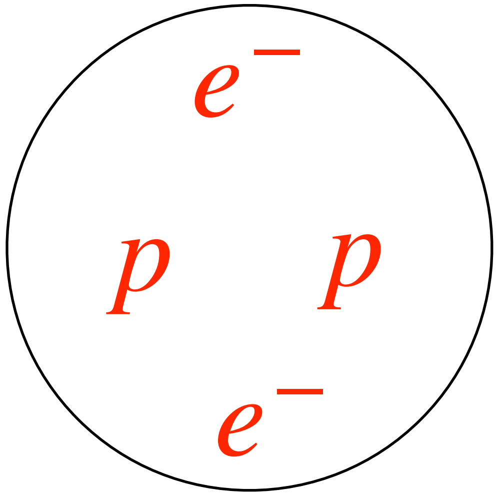
- charm quark mass ? light quark mass ? α_s ?
- string tension σ ? ($V(r) \to \sigma\, r$ at large *r*)
- string-breaking transition rate (from creation of light $q\bar{q}$ pair) ?

H dimer

Interatomic potential for H atoms:
one S-wave bound state



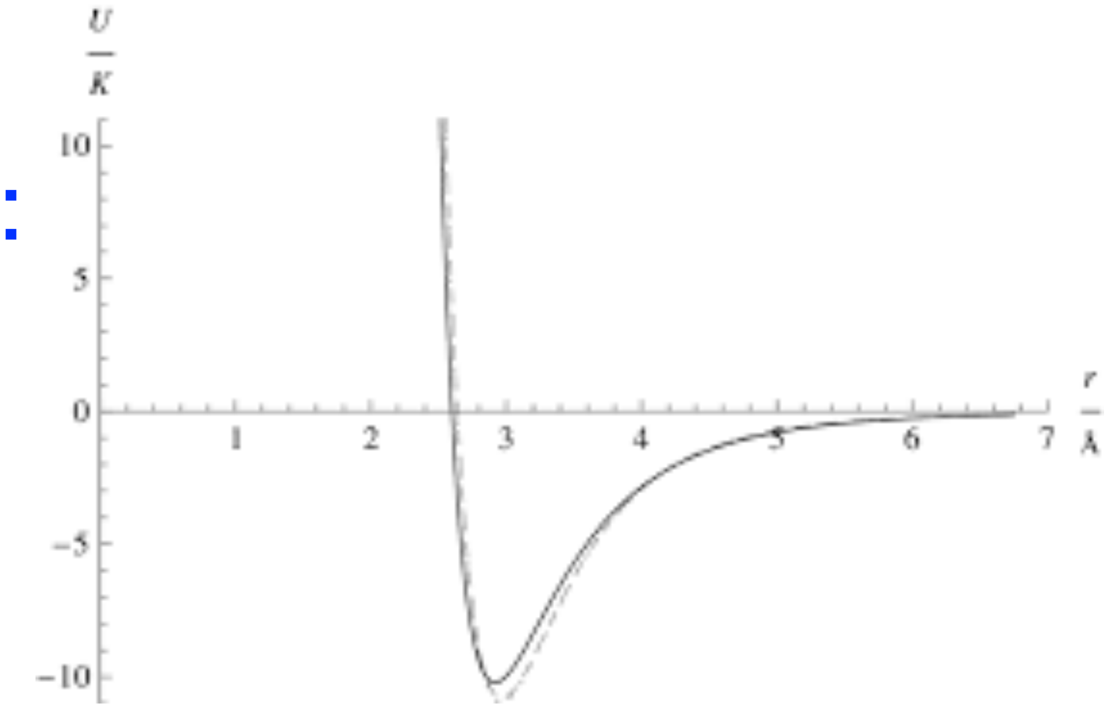
H diatomic molecule:
2 protons and 2 electrons



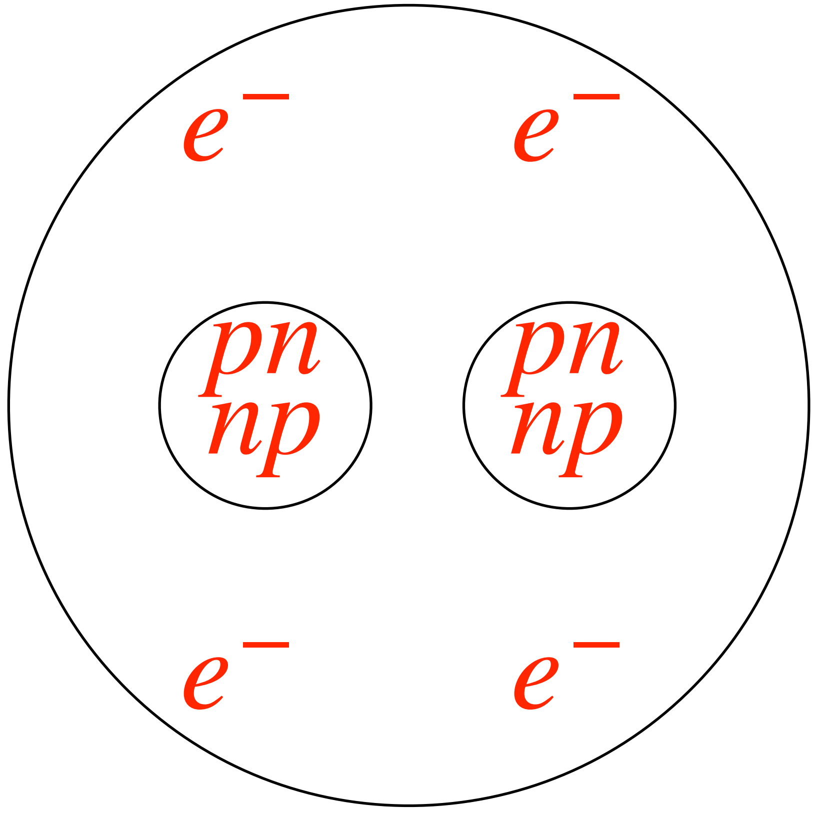
binding energy: 4.7 eV
bond length: 0.074 nm

He-4 dimer

Interatomic potential for ^{4}He atoms:
near critical depth
for 1st S-wave bound state

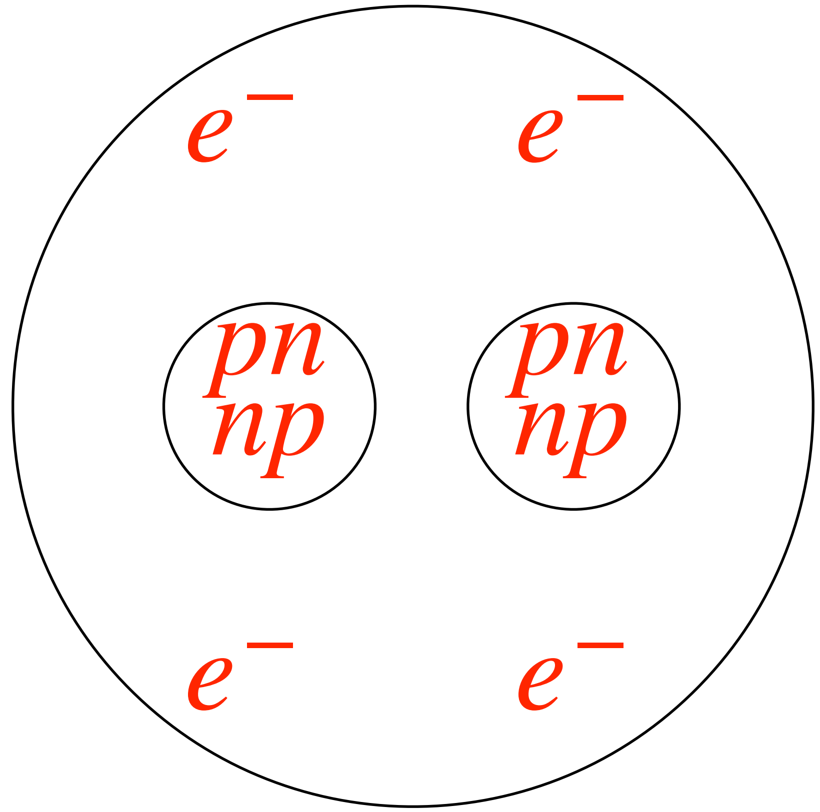


^{4}He diatomic molecule:
2 ^{4}He nuclei and 4 electrons



Is it bound?
What is its nature?

He-4 dimer



1st experimental observation Schollkopf & Toennies (1994)

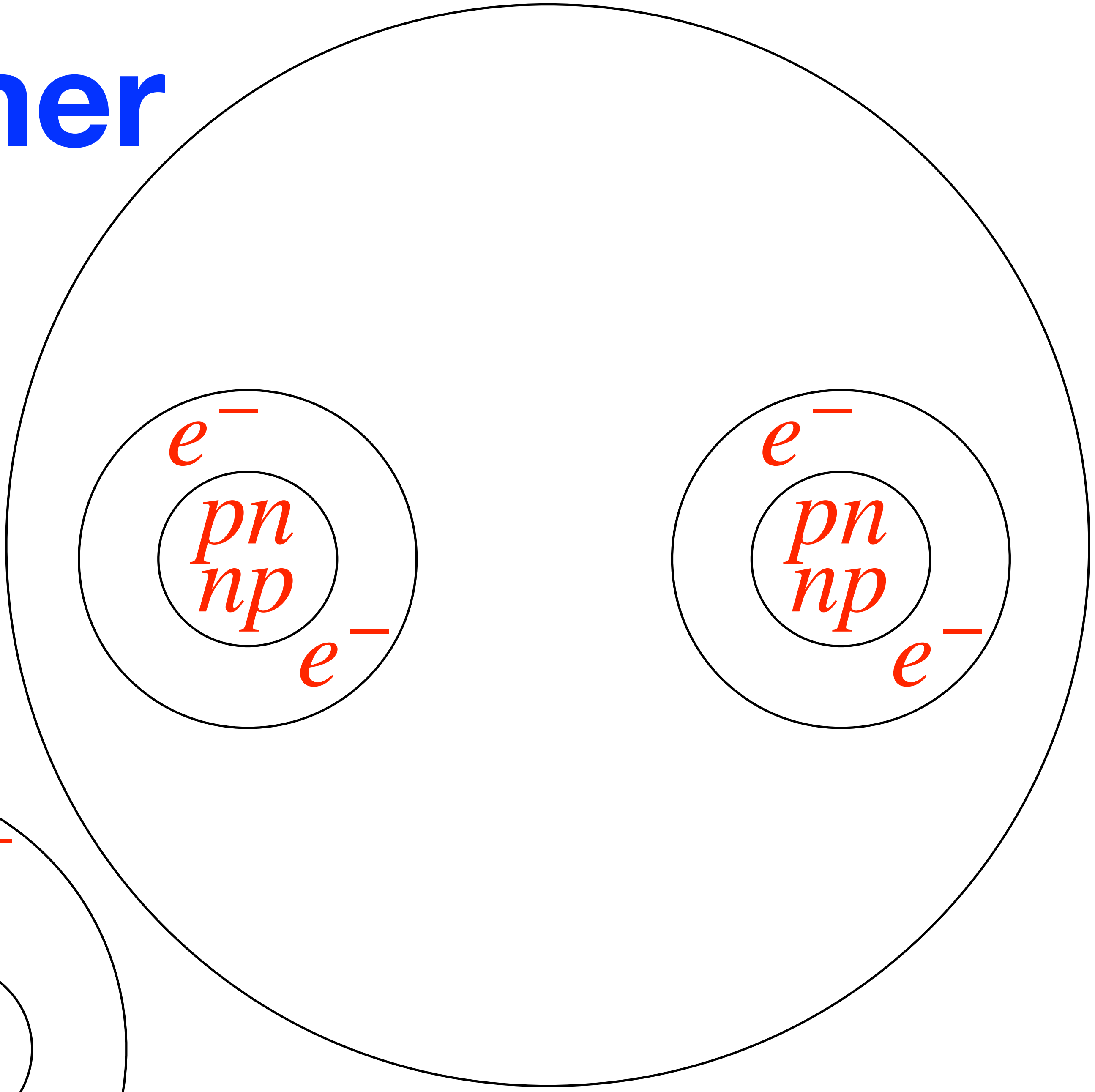
diffraction of ^{4}He beam from 100 nm transmission grating

measure bond length: 5.2 +/- 0.4 nm

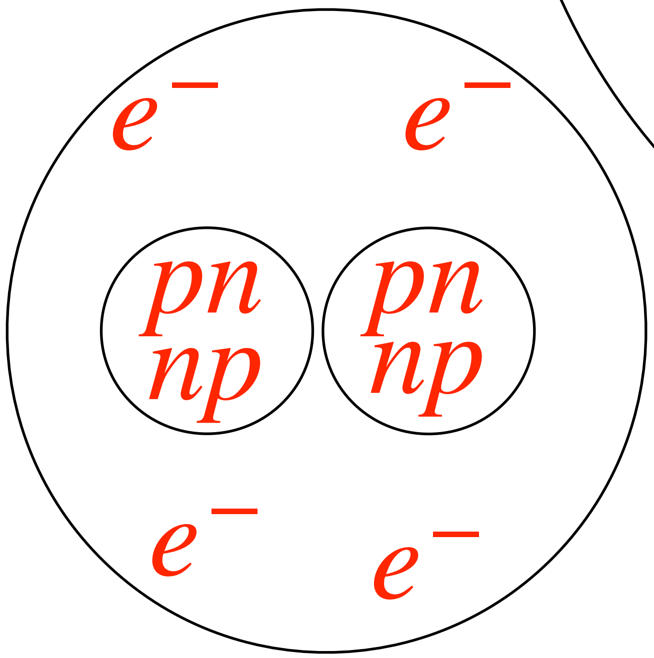
He-4 dimer

What is its nature ?

Is it a $(^4He)_2$ molecule ?



Is it a ^8Be atom ?



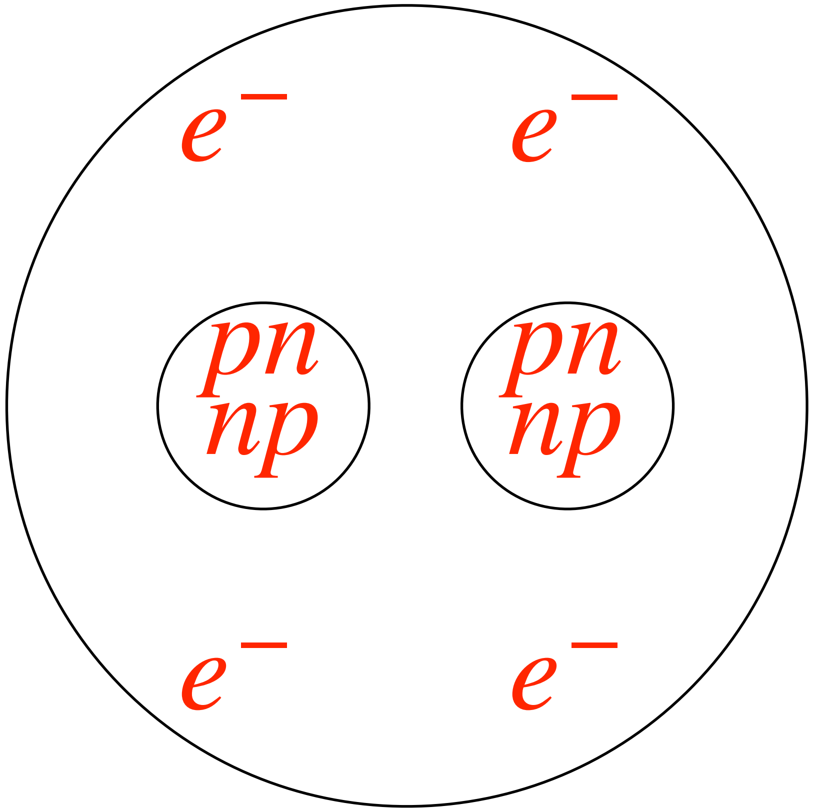
Measurement of bond length
resolved issue experimentally in favor of $(^4He)_2$ molecule

He-4 dimer

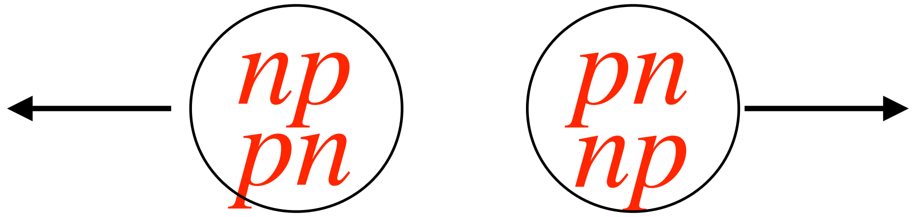
Coulomb explosion imaging using free electron laser

Zeller, …, & Dorner (2016)

He-4 dimer: 2 helium nuclei and 4 electrons



Use laser to blow away electrons



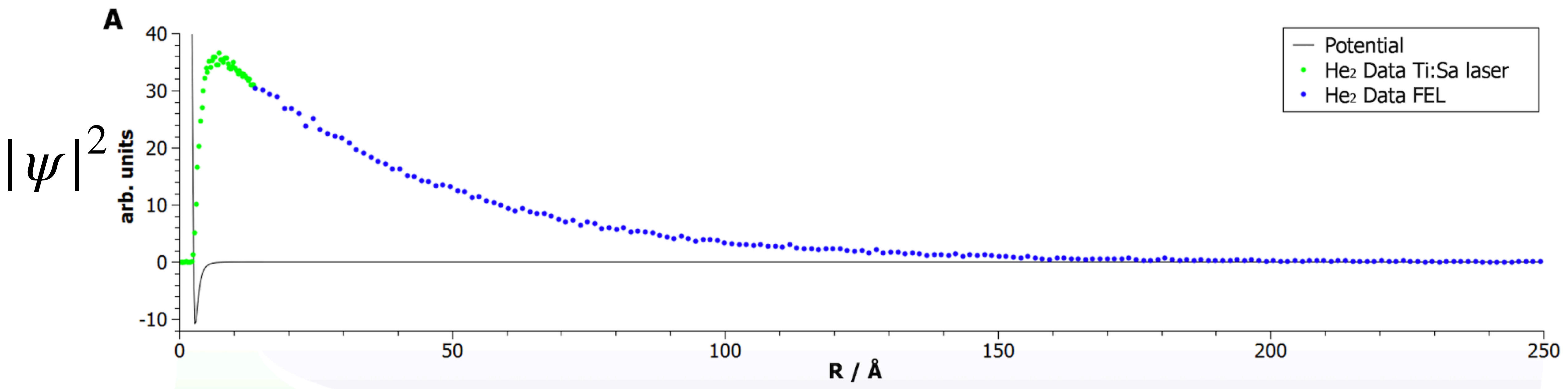
Track ^{4}He nuclei,
determine initial separation R

He-4 dimer

Coulomb explosion imaging using free electron laser

Zeller, …, & Dorner (2016)

measure $|\psi|^2$ as function of *R* up to 250 Angstroms



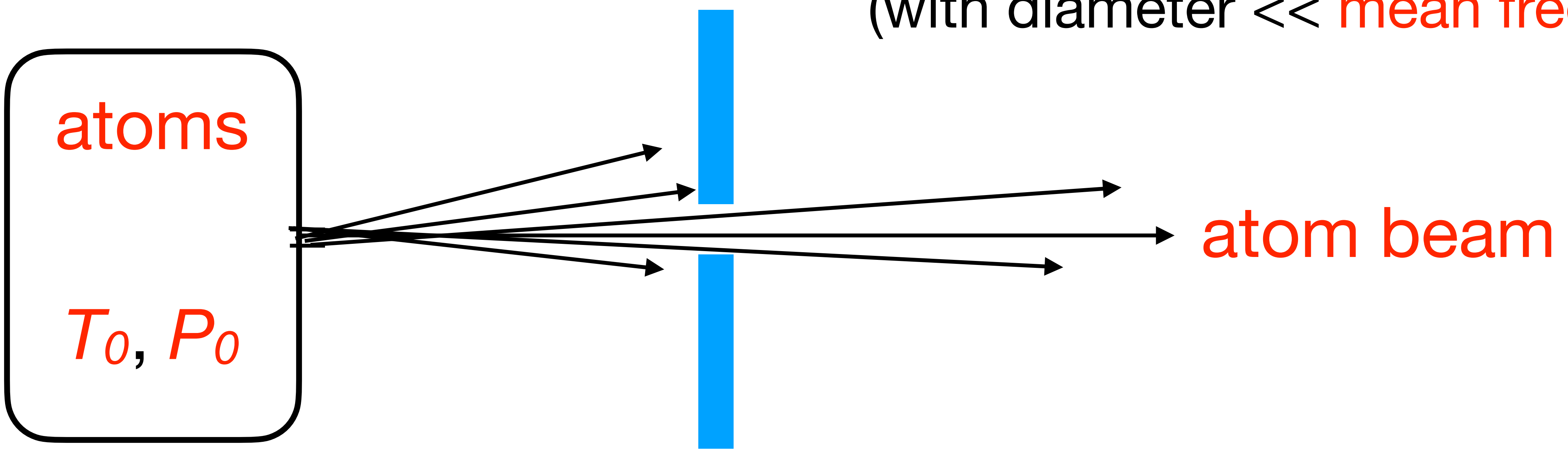
deduce binding energy: $|\varepsilon| = (1.52 \pm 0.13) \times 10^{-7}$ eV

Atomic Beam from Free Expansion into Vacuum

Bottle of atoms at temperature T_0, pressure P_0

Atoms escape into vacuum through small nozzle

(with diameter << mean free path)



Create beam by blocking atoms except along z axis

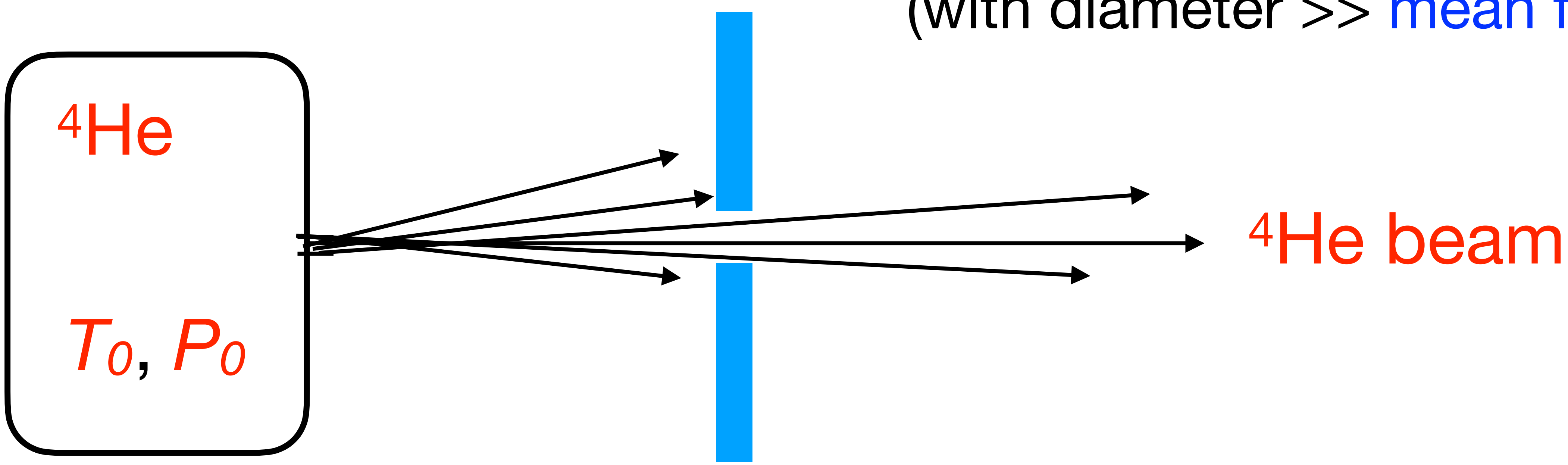
Momentum distribution of beam same as $p_z > 0$ inside bottle

^{4}He Beam from Free Expansion into Vacuum

Bottle of ^{4}He atoms at temperature T_0, pressure P_0

Atoms escape into vacuum through small nozzle

(with diameter >> mean free path)



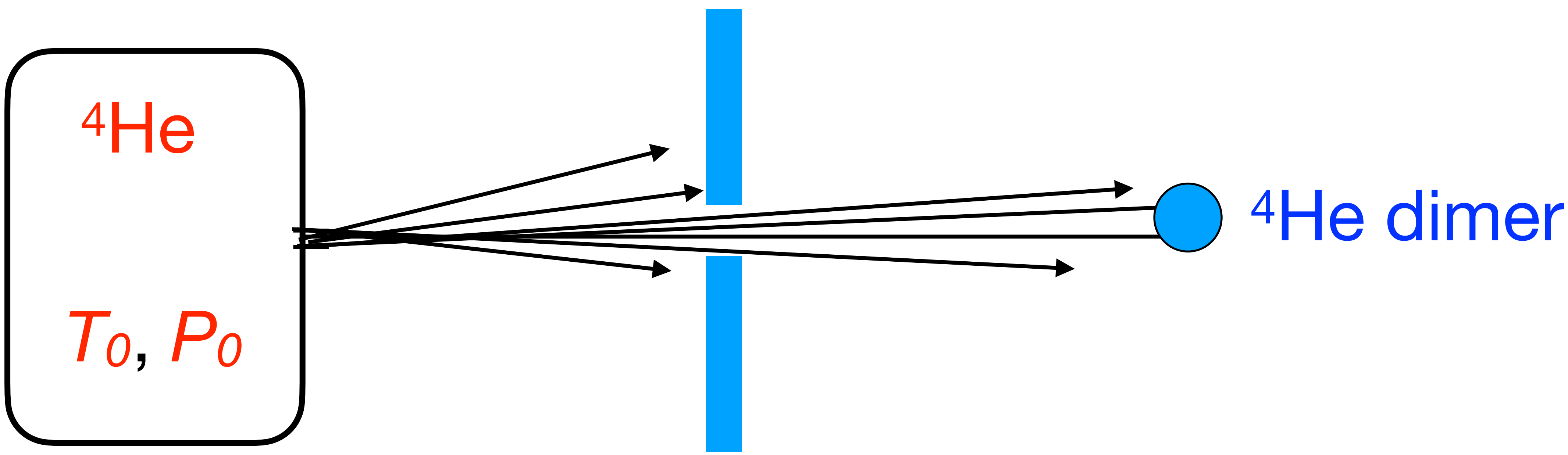
Atoms along *z* axis in local equilibrium until kinetic freezeout
temperature *T(z)*, pressure *P(z)*, mean speed *u(z)*

Pressure accelerates atoms to supersonic speed: $u(z) \gg \sqrt{\frac{5kT}{3m}}$

^{4}He Dimers from Free Expansion into Vacuum

Bottle of ^{4}He atoms at temperature T_0, pressure P_0
Atoms escape into vacuum through small nozzle

(with diameter >> mean free path)



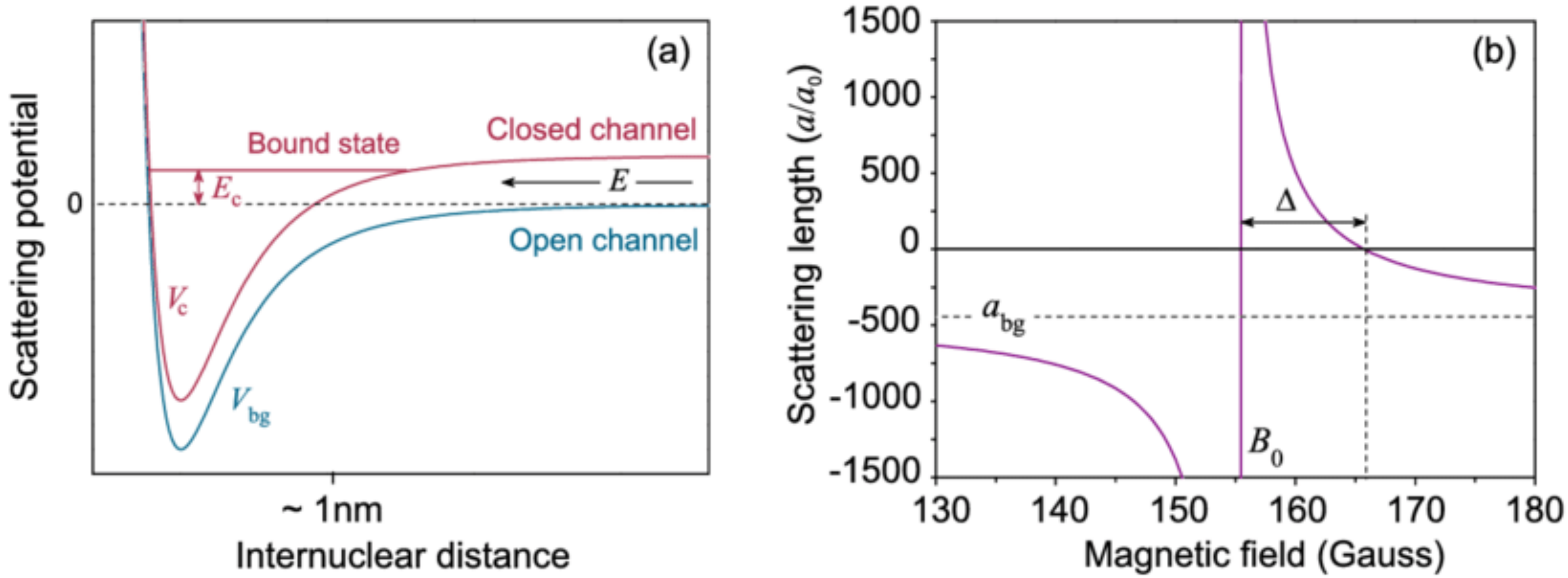
Atoms along *z* axis in local equilibrium until kinetic freezeout
temperature *T(z)*, pressure *P(z)*, mean speed *u(z)*

What is formation rate of ^{4}He dimer as function of T_0, P_0?

Magnetic Feshbach Resonance

Two interatomic potentials with different magnetic moments

Magnetic field B can tune bound state in “closed channel” to threshold of “open channel”



Magnetic field B can tune scattering length in open channel

$$a(B) = a_{bg} + \frac{\Delta}{B - B_0}$$

$\Longrightarrow$ controllable large scattering length

unitary limit: $B \longrightarrow B_0$

Fermi Gas with Large Scattering Length

Fermionic atoms with two spin states $\sigma = 1{,}2$

trapped in 3D harmonic potential: $V(r) = \frac{1}{2} m\omega^2 r^2$

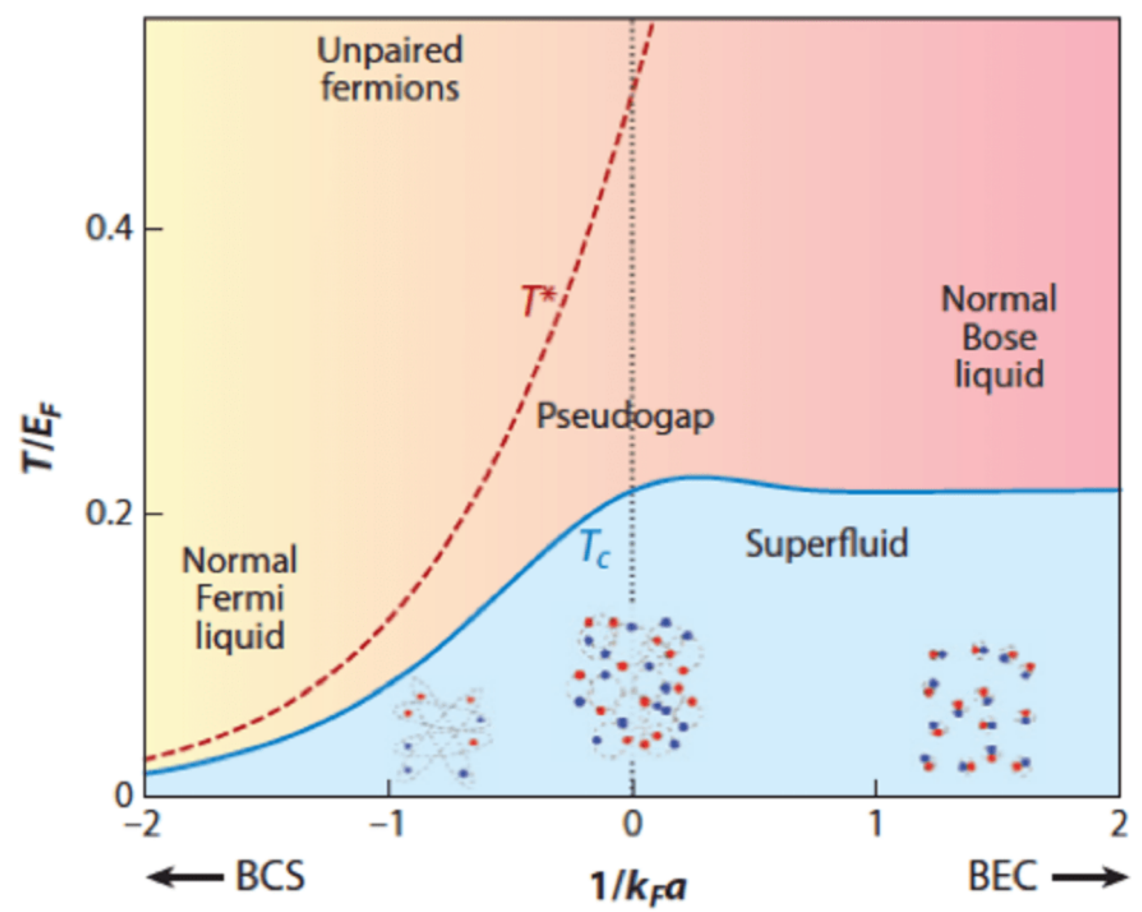
cooled to ultralow temperatures

scattering length *a(B)* controlled using magnetic Feshbach resonance

Superfluid below critical temperature that depends on *a*

smooth crossover as function of *1/a*

- *1/a < 0:* BCS superfluid (Cooper pairing of fermions)
- *1/a = 0:* unitary superfluid (*??*)
- *1/a > 0:* BEC superfluid (Bose-Einstein condensation of loosely bound molecules)



Contact for Fermi Gas

In 2003, Shina Tan derived several surprising results
for the Fermi gas with large scattering length
that involved a quantity called the contact *C*

- Momentum distribution has power-law tail at large *k*
  with same coefficient *C* for $\sigma = 1,2$:
  $$n_\sigma(k) = C/k^4$$

- Adiabatic relation: $C = -4\pi m \left(\frac{\partial E}{\partial(1/a)} \right)_S$

  $\Longrightarrow$ *C* is essentially thermodynamic variable conjugate to *1/a*

- Pressure relation for homogeneous system:
  $$\mathcal{P} = \frac{2}{3}\mathcal{E} + \frac{1}{12\pi m a}\mathcal{C}$$
  $\mathcal{C}$ is contact density

Many other universal relations involving the contact have been derived.
Some have been verified experimentally.

Contact for Loosely Bound Molecule

energy of molecule: $\varepsilon = -\,1/(m\,a^2)$

Contact of molecule: $C = -\,4\pi m \dfrac{\partial \varepsilon}{\partial(1/a)} = \dfrac{8\pi}{a}$

<u>Many-body system</u>

Low-density limit: contact comes entirely from molecules:

$\mathscr{C} \longrightarrow n_M\,(8\pi/a)$, where n_M is molecule density

Strategy for calculating formation rate of loosely bound molecule

- Calculate contact density $\mathscr{C}$ in equilibrium as function of *T, P*
- Determine contact density $\mathscr{C}$ at kinetic freezeout
- Deduce molecule density after free streaming to low density

$$\mathscr{C} \longrightarrow n_M\,(8\pi/a)$$

Summary

A loosely bound molecule

has universal properties determined by its binding energy,
including size much larger than interaction range of its constituents

In atomic physics,

there is definitive experimental proof
of the large sizes of loosely bound molecules.

In hadronic physics,

there will be no such definitive experimental proof.
The best one can do is show that loosely bound molecules
provide simpler explanations of their properties.

Insights from atomic physics on loosely bound molecules
could continue to be useful for loosely bound hadronic molecules.