# Introduction to Effective Field Theory

## Lecture 1: EFTs for QED


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# Quantum Field Theory

## Traditional QFT

application of Quantum Mechanics to fields for elementary particles:
electromagnetic fields,
Dirac field for electron, …

## General principles:

quantum mechanics
special relativity
cluster decomposition
(AND indistinguishability of identical bosons or identical fermions)

## Ultimate goal:

describe elementary particles to arbitrarily high energy
with arbitrarily high accuracy
using finite number of parameters

## Renormalizability

Renormalizable QFT's, such as QED, have finite number of parameters
If you add any additional operator to the Lagrangian,
renormalization will generate all the infinitely many operators
consistent with the symmetries

# Quantum Field Theory

## Stephen Weinberg’s folk theorem (1979)

“If one writes down the most general possible Lagrangian,
including all terms consistent with assumed symmetry principles,
and then calculates matrix elements with this Lagrangian
to any given order of perturbation theory,
the result will simply be the most general possible S-matrix
consistent with perturbative unitarity,
analyticity,
cluster decomposition,
and the assumed symmetry properties.”

Such a Lagrangian will include infinitely many terms
(including ones with arbitrarily high derivatives).
For this to be a useful basis for a legitimate dynamical theory,
it is necessary that in some sort of perturbative expansion,
only a finite number of terms in the Lagrangian can appear
in each order of perturbation theory.

# Quantum Field Theory

## Weinberg's Effective Field Theory Philosophy

QFT should describe particles only up to some high energy scale $\Lambda$
(set by masses of lightest particles not described explicitly)

- systematically improvable accuracy
  by expanding to higher order in energy/$\Lambda$
- infinitely many parameters but finite number at any given order in the energy expansion

## High-energy EFT

model-independent framework for higher energy physics

## Low-energy EFT

systematic approximations for lower energy physics

# Quantum ElectroDynamics

QFT for electrons positrons and photons

electromagnetic gauge field $A_\mu(x)$

Dirac spinor field for electron $\psi(x)$

electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

covariant derivative $D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi$

Lagrangian $\mathscr{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu \psi - m_e \overline{\psi}\psi$

2 parameters: $\alpha = \dfrac{e^2}{4\pi} = \dfrac{1}{137.036}$ $\quad m_e = 0.511$ MeV

<u>Continuous symmetries</u>

Lorentz invariance

$U(1)$ gauge invariance

<u>Discrete symmetries</u>

Parity $P$

Charge conjugation $C$

# Quantum ElectroDynamics

Lagrangian $\mathscr{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu\psi - m_e\overline{\psi}\psi$

Continuous symmetries

$U(1)$ gauge invariance

Lorentz invariance

Discrete symmetries

Parity $P$

Charge conjugation $C$

Approximate chiral symmetry: $U(1)_L \times U(1)_R$

becomes increasingly accurate in the limit $m_e \to 0$

(but not exact: $U(1)_A$ broken by triangle anomaly at order $\alpha$)

Chiral components of Dirac spinor field:

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi\,, \quad \psi_R = \frac{1}{2}(1+\gamma_5)\psi$$

$$\mathscr{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}_L\gamma^\mu D_\mu\psi_L + i\overline{\psi}_R\gamma^\mu D_\mu\psi_R - m_e(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$$

helicity conservation violated by terms proportional to $m_e$



Approximate scale invariance ($\Longrightarrow$ conformal symmetry)

broken by renormalization of $\alpha$ at order $\alpha^2$

# Quantum ElectroDynamics

QED is perturbatively renormalizable

Reaction rates for photons, electrons, and positrons can be calculated to increasingly higher precision
(but not increasingly higher accuracy)
as expansions in powers of $\alpha$

Magnetic moment of electron

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi} - 0.657\left(\frac{\alpha}{\pi}\right)^2 + 1.181\left(\frac{\alpha}{\pi}\right)^3 - 1.912\left(\frac{\alpha}{\pi}\right)^4 + 6.737\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

↑ no further improvement in accuracy

Problem

other charged particles besides $e^{\pm}$: $\mu^{\pm}, \pi^{\pm}, p, \bar{p}, \ldots$
QED is not a complete theory of electrodynamics

# Traditional QFT

Ultimate Goal

describe all elementary particles up to arbitrarily high energy
with arbitrarily high accuracy
using finite number of parameters

Intermediate goal QFT that includes QED
but describes more elementary particles

- QED with muons

  additional parameter: $m_\mu$

  correction to $g/2$: $2.8 \times 10^{-12}$ from $\alpha^2 (m_e/m_\mu)$ term

- QED + QCD (with light quarks and gluons)

  4 additional parameters: $\alpha_s$ and 3 quark masses

  correction to $g/2$: $1.7 \times 10^{-12}$ from QCD interactions

- Standard Model of Particle Physics

  total of 18 parameters

  correction to $g/2$: $0.03 \times 10^{-12}$ from weak interactions

prediction for $g/2$ agrees with measured value to within experimental error $0.3 \times 10^{-12}$

# Quantum ElectroDynamics

## Scaling dimensions of operators

derivative: $\partial_\mu$ has scaling dimension 1

Determine scaling dimension of field
by requiring kinetic term to have scaling dimension = 4 (spacetime dimension)

gauge field: $-\frac{1}{4}\partial_\mu A_\nu \partial^\mu A^\nu \Longrightarrow A_\mu$ has scaling dimension $(4-2)/2 = 1$

Dirac field: $i\overline{\psi}\gamma^\mu\partial_\mu\psi \Longrightarrow \psi$ has scaling dimension $(4-1)/2 = 3/2$

electron mass term: $-m_e\overline{\psi}\psi$ operator $\overline{\psi}\psi$ has scaling dimension 3

QED interaction term: $-eA_\mu\overline{\psi}\gamma^\mu\psi$ operator $A_\mu\overline{\psi}\gamma^\mu\psi$ has scaling dimension 4

QED Lagrangian: $\mathscr{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu\psi - m_e\overline{\psi}\psi$

includes all terms with scaling dimensions $d \leq 4$

consistent with continuous symmetries: Lorentz invariance, $U(1)$ gauge invariance

discrete symmetries: parity $P$, charge conjugation $C$

renormalizable, two real parameters: $\alpha$, $m_e$

# Low-energy Effective QED

Euler-Heisenberg effective field theory (Quantum PhotoDynamics)
describes photons up to an energy scale $m_e$
with increasingly higher precision by expanding in powers of energy/$m_e$

electromagnetic gauge field $A_\mu(x)$

electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Lagrangian $\mathscr{L}_{\mathrm{QPhD}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{d \geq 6} \frac{1}{m_e^{d-4}} \sum_n c_{d,n} \mathcal{O}_{d,n}$

Leading order Lagrangian: $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

- describes free photons
- no parameters

## Higher dimension operators

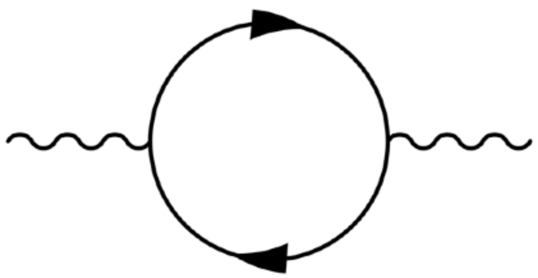
consistent with Lorentz invariance, $U(1)$ gauge symmetry, discrete symmetries $P$ and $C$

- sum over scaling dimensions: $d = 6, 8, 10, \ldots,$
- for each scaling dimension $d$, sum over independent operators $\mathcal{O}_{d,n}$
- dimensionless coefficients $c_{d,n}$: calculate from QED as expansions in powers of $\alpha$

# Low-energy EFT for QED

describes photons up to an energy scale $m_e$

## Integrate out electron loops $m_e$



Expand in powers of photon 4-momentum $q^\mu$ divided by $m_e$

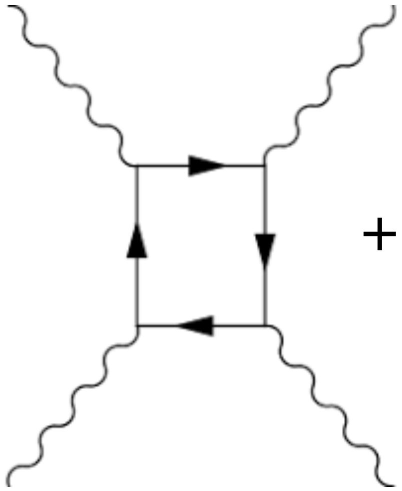
0th order: vanishes by gauge invariance

2nd order: matches vertex from operator $F_{\mu\nu}F^{\mu\nu}$

(absorbed into definition of $\alpha$)

4th order: matches vertices from operators of form $F^2\partial^2$

(redundant operators)



+ 5 permutations of legs

Expand in powers of photon 4-momenta divided by $m_e$

0th, 2nd order: cancels by gauge invariance

4th order: matches vertices from operators of form $F^4$

(scattering of light by light)

6th order; matches vertices from operators of form $F^4\partial^2$

# Low-energy EFT for QED

Effective field theory for photons up to energy scale $m_e$

increasingly higher precision by expanding in powers of energy/$m_e$

Lagrangian $\mathscr{L}_{\rm QPhD} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{d\geq 6} \frac{1}{m_e^{d-4}} \sum_n c_{d,n}\mathcal{O}_{d,n}$

$U(1)$ gauge invariance $\Longrightarrow$ operators can depend on $A_\mu$ only through $F_{\mu\nu}$

Charge conjugation symmetry: $A_\mu(x) \longrightarrow -A_\mu(x) \Longrightarrow F_{\mu\nu} \longrightarrow -F_{\mu\nu}$

$\Longrightarrow$ operators must have even number of factors of $F_{\mu\nu}$

Lorentz invariance: operators must be scalars $\Longrightarrow$ even number of factors of $\partial_\mu$

schematic forms of operators: $F^{2m}\partial^{2n}$, $d = 4m + 2n$

## Higher dimension operators

- for each scaling dimension $d$, sum over independent operators $\mathcal{O}_{d,n}$ consistent with symmetries
- dimension-6 operators: $FF\partial\partial$
  5 ways to contract 6 indices
- dimension-8 operators: $FFFF$ or $FF\partial\partial\partial\partial$
  many ways to contract 8 indices

# Quantum ElectroDynamics

Effective field theory for photons up to energy scale $m_e$

Lagrangian $\mathscr{L}_{\text{QPhD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathscr{L}_6 + \mathscr{L}_8 + \ldots$

## Dimension-6 operators

schematic form: $FF\partial\partial$

5 distinct operators: $F_{\mu\nu}\partial^2 F^{\mu\nu}$, $\partial_\alpha F_{\mu\nu}\partial^\alpha F^{\mu\nu}$, $\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$, $\partial_\alpha F_{\mu\beta}\partial^\beta F^{\mu\alpha}$, $F_{\mu\alpha}\partial^\alpha\partial_\beta F^{\mu\beta}$

but some of the operators are redundant

Permutation identity $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \implies \partial_\alpha F_{\mu\nu} + \partial_\mu F_{\nu\alpha} + \partial_\nu F_{\alpha\mu} = 0$

$\partial_\alpha F_{\mu\nu}\partial^\alpha F^{\mu\nu} = -\,\partial_\alpha F_{\mu\nu}(\partial^\nu F^{\alpha\mu} + \partial^\mu F^{\nu\alpha}) = 2\,\partial_\alpha F_{\mu\beta}\partial^\beta F^{\mu\alpha} \implies \partial_\alpha F_{\mu\nu}\partial^\alpha F^{\mu\nu}$ is redundant

$\partial_\alpha F_{\mu\beta}\partial^\beta F^{\mu\alpha} = -\,\partial_\alpha F_{\mu\beta}(\partial^\mu F^{\alpha\beta} + \partial^\alpha F^{\beta\mu}) = -\,\partial_\alpha F_{\mu\beta}\partial^\beta F^{\mu\alpha} + \partial_\alpha F_{\mu\beta}\partial^\alpha F^{\mu\beta} \implies \partial_\alpha F_{\mu\beta}\partial^\beta F^{\mu\alpha}$ is redundant

Redundant operators from Integration By Parts (IBP)

terms in the Lagrangian that are total derivatives: $\partial_\mu \Phi^\mu$

do not contribute to the action: $\int d^4x\,\partial_\mu\Phi^\mu = 0$

and therefore do not contribute to the path integral



$F_{\mu\nu}\partial^2 F^{\mu\nu} = \partial_\alpha(F_{\mu\nu}\partial^\alpha F^{\mu\nu}) - (\partial_\alpha F_{\mu\nu})\partial^\alpha F^{\mu\nu} \implies F_{\mu\nu}\partial^2 F^{\mu\nu}$ is redundant

$F_{\mu\alpha}\partial^\alpha(\partial_\beta F^{\mu\beta}) = \partial^\alpha(F_{\mu\alpha}\partial_\beta F^{\mu\beta}) - (\partial^\alpha F_{\mu\alpha})\partial_\beta F^{\mu\beta} \implies F_{\mu\alpha}\partial^\alpha\partial_\beta F^{\mu\beta}$ is redundant

# Quantum ElectroDynamics

Effective field theory for photons up to energy scale $m_e$

Lagrangian $\mathscr{L}_{\text{QPhD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathscr{L}_6 + \mathscr{L}_8 + \dots$

Equations Of Motion from $d = 4$ term: $\partial_\mu F^{\mu\nu} = 0$

## Dimension-6 operators

schematic form: $FF\partial\partial$

Redundant operators from Equations of Motion (EOM)

path integral is invariant under redefinition of $A_\mu$ field: $A_\mu \longrightarrow A_\mu + \Phi_\mu$

any term in the Lagrangian proportional $d = 4$ EOM

can be canceled by redefinition of $A_\mu$ field: $A_\mu \longrightarrow A_\mu + \Phi_\mu$

$$S[A] \longrightarrow S[A + \phi] = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F^{\mu\nu}\partial_\mu\Phi_\nu + \mathscr{L}_6 + \dots\right)$$

$$= \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu F^{\mu\nu}\Phi_\nu + \mathscr{L}_6 + \dots\right)$$

a higher dimension term in $\mathscr{L}_6$, $\mathscr{L}_8$, … with factor of $\partial_\mu F^{\mu\nu}$

can be canceled by appropriate choice of $\Phi_\nu$

$\Longrightarrow$ its effects enter at higher order in the energy expansion than naively expected

# Quantum ElectroDynamics

Effective field theory for photons up to energy scale $m_e$

Lagrangian $\mathscr{L}_{\text{QPhD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathscr{L}_6 + \mathscr{L}_8 + \ldots$

Equations of Motion from $d = 4$ term: $\partial_\mu F^{\mu\nu} = 0$

## Dimension-6 operators

5 distinct operators with schematic form $FF\partial\partial$

After removing redundant operators from permutation identities and IBP identities,

there is only one dimension-6 operator: $\mathscr{L}_6 = \frac{c_6}{m_e^2}\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$

Redundant operators from Equations of Motion (EOM)

higher dimension operators with factor of $\partial_\mu F^{\mu\nu}$

can be canceled by redefinition of $A_\mu$ field: $A_\mu \longrightarrow A_\mu + \Phi_\mu$

choose $\Phi_\nu = -(c_6/m_e^2)\partial^\lambda F_{\lambda\nu}$

$$S[A] \longrightarrow S[A+\Phi] = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu F^{\mu\nu}\Phi_\nu + \mathscr{L}_6 + \mathscr{L}_8 + \ldots\right)$$

$$= \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathscr{L}_8 + \ldots\right)$$



$\Longrightarrow$ all operators of dimension 6 are redundant

# Quantum ElectroDynamics

Effective field theory for photons up to energy scale $m_e$

Lagrangian $\mathscr{L}_{\text{QPhD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathscr{L}_8 + \mathscr{L}_{10} + \mathscr{L}_{12} + \ldots$

## Dimension-8 operators

schematic forms: $F^4$ or $F^2\partial^4$

many ways to contract 8 indices

many redundant operators from permutation identities
from Integration By Parts
from Equations Of Motion

only two independent operators:

$$\mathscr{L}_8 = -\frac{5\pi\,\alpha^2}{180\,m_e^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{14\pi\,\alpha^2}{180\,m_e^4}F_{\mu\nu}F^{\nu\lambda}F_{\lambda\sigma}F^{\sigma\mu}$$

## Dimension-10 operators

3 operators of the form $F^4\partial^2$

no operators of the form $F^2\partial^6$

## Dimension-12 operators

2 operators of the form $F^6$

3 operators of the form $F^4\partial^4$

no operators of the form $F^2\partial^8$

operators can be enumerated using Hilbert series

# High-energy EFT for QED

Describes photons, electrons, and positrons up to a high energy scale $\Lambda$

($\Lambda$ is greater than $m_e$ but at most of order $m_\mu$ or $m_\pi$)

to increasingly higher precision by expanding in powers of $\alpha$

and in powers of energy/$\Lambda$

<u>QED Lagrangian</u> $\mathscr{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu \psi - m_e \overline{\psi}\psi$

<u>Effective field theory</u> $\mathscr{L}_{\text{EFT}} = \mathscr{L}_{\text{QED}} + \sum_{d\geq 5} \frac{1}{\Lambda^{d-4}} \sum_n c_{d,n}\mathscr{O}_{d,n}$

- sum over scaling dimensions: $d = 5,6,7,8,\ldots,$
- for each scaling dimension $d$, sum over independent operators $\mathscr{O}_{d,n}$
  consistent with Lorentz invariance
  and $U(1)$ gauge invariance
  (but not discrete symmetries $P$ and $C$)
- dimensionless coefficients $c_{d,n}$: calculate from more fundamental theory
  (or treat as phenomenological parameters)

infinitely many parameters: $\alpha$, $m_e$

$c_{5,1}$

$c_{6,1}$, $c_{6,2}$, $c_{6,3}$, …

…

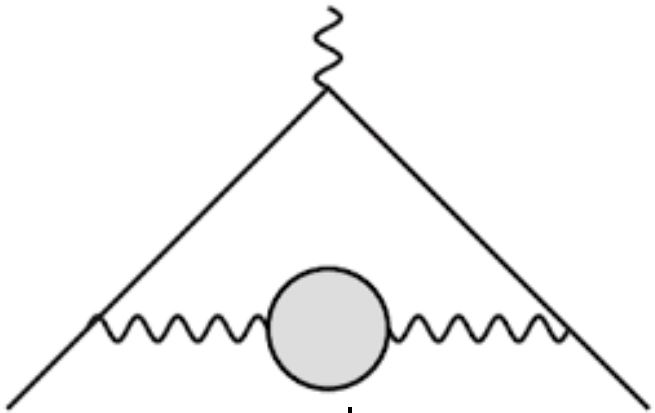
but finite number at any order in energy/$\Lambda$

# High-energy EFT for QED

describe photons, electrons, and positrons up to energy scale $\Lambda$ of order $m_\mu$

Lagrangian $\mathscr{L}_{\text{QEDEFT}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu\psi - m_e\overline{\psi}\psi + \mathscr{L}_5 + \ldots$

Dimension-5 operator $\mathscr{L}_5 = -\dfrac{fe}{\Lambda}F_{\mu\nu}\overline{\psi}\sigma^{\mu\nu}\psi$




operator breaks $U(1)_L \times U(1)_R$ chiral symmetry

so matrix element must have factor of $m_e$

loop involves charged particles, so coefficient $f$ must have factor of $(\alpha/\pi)^2$

contributions from $\mu^\pm$, hadrons, weak bosons, $\tau^\pm$, …

$$\frac{f}{\Lambda} = \frac{f_\mu + f_{\text{had}} + f_{\text{weak}} + f_\tau + \ldots}{\Lambda} = \frac{(2.7 + 1.7 + 0.03 + 0.01 + \ldots) \times 10^{-5}}{m_\mu}\left(\frac{\alpha}{\pi}\right)^2$$



Contributions to electron magnetic moment

$$\Delta(g/2) = 4\,(f_\mu + f_{\text{had}} + f_{\text{weak}} + f_\tau + \ldots) = (2.8 + 1.7 + 0.03 + 0.01 + \ldots) \times 10^{-12}$$

# High-energy EFT for QED

describe photons, electrons, and positrons up to energy scale $\Lambda$ of order $m_\mu$

Lagrangian $\mathscr{L}_{\mathrm{QEDEFT}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu\psi - m_e\overline{\psi}\psi + \mathscr{L}_5 + \mathscr{L}_6 + \dots$

## Dimension-6 operators that respect Parity

schematic forms: $FF\partial\partial$, $(\overline{\psi}\Gamma\psi)(\overline{\psi}\Gamma\psi)$

- 5 $FF\partial\partial$ operators: all redundant by permutation identities,
  Integration By Parts,
  and Equations Of Motions

- $(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma^\mu\psi)$, $(\overline{\psi}\gamma_\mu\gamma_5\psi)(\overline{\psi}\gamma^\mu\gamma_5\psi)$
  respects $U(1)_L \times U(1)_R$ chiral symmetry

- $(\overline{\psi}\psi)(\overline{\psi}\psi)$, $(\overline{\psi}\sigma_{\mu\nu}\psi)(\overline{\psi}\sigma^{\mu\nu}\psi)$, $(\overline{\psi}\gamma_5\psi)(\overline{\psi}\gamma_5\psi)$
  breaks $U(1)_L \times U(1)_R$ chiral symmetry so matrix element must have factor of $m_e$

# High-energy EFT for QED

describe photons, electrons, and positrons up to an energy scale $\Lambda$ of order $m_\mu$

to increasingly higher precision by expanding in powers of $\alpha$

and in powers of energy/$\Lambda$

QED $\mathscr{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi - m_e \bar{\psi}\psi$

EFT $\mathscr{L}_{\text{QEDEFT}} = \mathscr{L}_{\text{QED}} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \mathscr{L}_8 + \ldots$

Higher dimension operators that respect Parity

- Dimension 5: one operator $F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi$
  suppressed by $m_e/\Lambda$
- Dimension 6: 5 operators of the form $(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)$
  3 of them suppressed by $m_e/\Lambda$
- Dimension 7:
- Dimension 8:

operators can be enumerated using Hilbert series

# Standard Model

accurate description of all known elementary particles

## particles

gauge bosons: 8 gluons, $W^{\pm}, Z^0$, photon

$N_g = 3$ generations of fermions:

each with 2 quarks, 1 charged lepton, 1 left-handed neutrinos

Higgs boson

## Symmetries

$SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance

Lorentz invariance

no discrete symmetries imposed

(baryon number and lepton number conservation are consequences of renormalizability)

## Renormalizable QFT with 18* real parameters

$\mathscr{L}$ includes all operators with scaling dimensions $d \leq 4$ consistent with symmetries

3 gauge coupling constants

9 fermion masses and 4 quark mixing angles (from dimensionless Yukawa couplings)

Higgs self coupling



Higgs vacuum expectation value (only dimensional parameter)

* or 19 parameters if you count QCD CP-violation parameter $\theta$

# Standard Model EFT

High energy EFT for Standard Model
Model independent framework for physics beyond the Standard Model

<u>SMEFT Lagrangian:</u> $\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{d\leq 4} + \sum_{d\geq 5} \frac{1}{\Lambda^{d-4}} \sum_{n} c_{d,n} \mathscr{O}_{d,n}$

## Standard Model Lagrangian: $\mathscr{L}_{d\leq 4}$

- includes all terms with scaling dimensions $d \leq 4$
  consistent with Lorentz invariance
  and $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance
- 18 parameters

## Higher dimension operators

- sum over scaling dimensions: $d = 5,6,7,8,\ldots,$
- for each scaling dimension $d$, sum over independent operators $\mathscr{O}_{d,n}$
  consistent with Lorentz invariance
  and $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance
- dimensionless coefficients $c_{d,n}$: treat as phenomenological parameters
  (or calculate from more fundamental theory)

# Standard Model EFT

SMEFT Lagrangian: $\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{d\leq 4} + \sum_{d\geq 5} \frac{1}{\Lambda^{d-4}} \sum_{n} c_{d,n} \mathcal{O}_{d,n}$

Standard Model Lagrangian: $\mathscr{L}_{d\leq 4}$

- $N_g = 1$: 8 parameters
- $N_g = 3$: 18 parameters

Dimension-5 operators

- $N_g = 1$: 2 operators (Weinberg, 1979)
- $N_g = 3$: 12 operators

provides Majorana masses and mixing angles for 3 left-handed neutrinos

Dimension-6 operators

- $N_g = 1$: 94 operators (Buchmuller & Wyler, 1986)
- $N_g = 3$: 3055 operators (Grzadowski, Iskrzynski, Misiak & Rosiek, 2010)

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Dimension-7 operators

- $N_g = 1$: 30 operators (Lehman & Martin, 2016)
- $N_g = 3$: 1542 operators (Henning, Lu, Melia & Murayama, 2017)

Dimension-8 operators

- $N_g = 1$: 993 operators (Henning, Lu, Melia & Murayama, 2017)
- $N_g = 3$: 44807 operators

Dimension-15 operators

- $N_g = 1$: 6,518,462 operators (Marinissen, Rahn & Waalewijn, 2020)
- $N_g = 3$: 8,023,911,776 operators

operators can be enumerated using Hilbert series and then constructed explicitly