Introduction to Effective Field Theory

Lecture 3: Chiral EFT for QCD

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Quantum ChromoDynamics

QFT for quarks, antiquarks, and gluons (and hadrons)

<u>fields</u>

$SU(3)$ gauge fields: $A^a_\mu(x)$

Dirac spinor fields for quarks: $q(x)$ 3 light flavors: *u, d, s*

chiral components: $q_L(x) = \frac{1}{2}(1 - \gamma_5)\, q(x)$

$$q_R(x) = \frac{1}{2}(1 + \gamma_5)\, q(x)$$

<u>parameters</u>

QCD coupling constant α_s (or Λ_{QCD} ~ few hundred MeV)

quark masses: $m_u < m_d \ll m_s < \Lambda_{\text{QCD}}$

<u>exact symmetries</u>

$SU(3)$ gauge invariance
Lorentz invariance
Parity
Charge conjugation*

* provided CP-violation angle θ of QCD is 0

Quantum ChromoDynamics

approximate symmetries of QCD

$SU(3)$ flavor symmetry

$$\begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix} \longrightarrow U_V \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix} \qquad U_V \in SU(3)$$

broken by small quark mass differences: $m_d - m_u \ll m_s - m_d < \Lambda_{\text{QCD}}$

$\Longrightarrow$ QCD spectrum consists of particles with similar masses in $SU(3)$ multiplets

almost equal masses in $SU(2)$ multiplets

$SU(3)_L \times SU(3)_R$ chiral symmetry

$$\begin{pmatrix} u_L(x) \\ d_L(x) \\ s_L(x) \end{pmatrix} \longrightarrow U_L \begin{pmatrix} u_L(x) \\ d_L(x) \\ s_L(x) \end{pmatrix}, \quad \begin{pmatrix} u_R(x) \\ d_R(x) \\ s_R(x) \end{pmatrix} \longrightarrow U_R \begin{pmatrix} u_R(x) \\ d_R(x) \\ s_R(x) \end{pmatrix} \qquad U_L, U_R \in SU(3)$$

broken by small quark masses: $m_u < m_d \ll m_s < \Lambda_{\text{QCD}}$

and spontaneously broken to $SU(3)$ flavor symmetry by QCD vacuum

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approximate symmetries of QCD

QCD vacuum state

invariant under $SU(3)$ flavor symmetry

$SU(3)_L \times SU(3)_R$ chiral symmetry is spontaneously broken

by condensation of quark-antiquark pairs with opposite helicities (Cooper pairs)

$$u_L\bar{u}_L,\ d_L\bar{d}_L,\ s_L\bar{s}_L$$
$$u_R\bar{u}_R,\ d_R\bar{d}_R,\ s_R\bar{s}_R$$

small quark masses bias q and $\bar{q}$ to have same flavor

expectation values in QCD vacuum state: $\langle \bar{u}_R u_L \rangle = \langle \bar{d}_R d_L \rangle = \Delta \approx \langle \bar{s}_R s_L \rangle$

but $\langle \bar{u}_R d_L \rangle = \langle \bar{u}_R s_L \rangle = 0$

Goldstone theorem

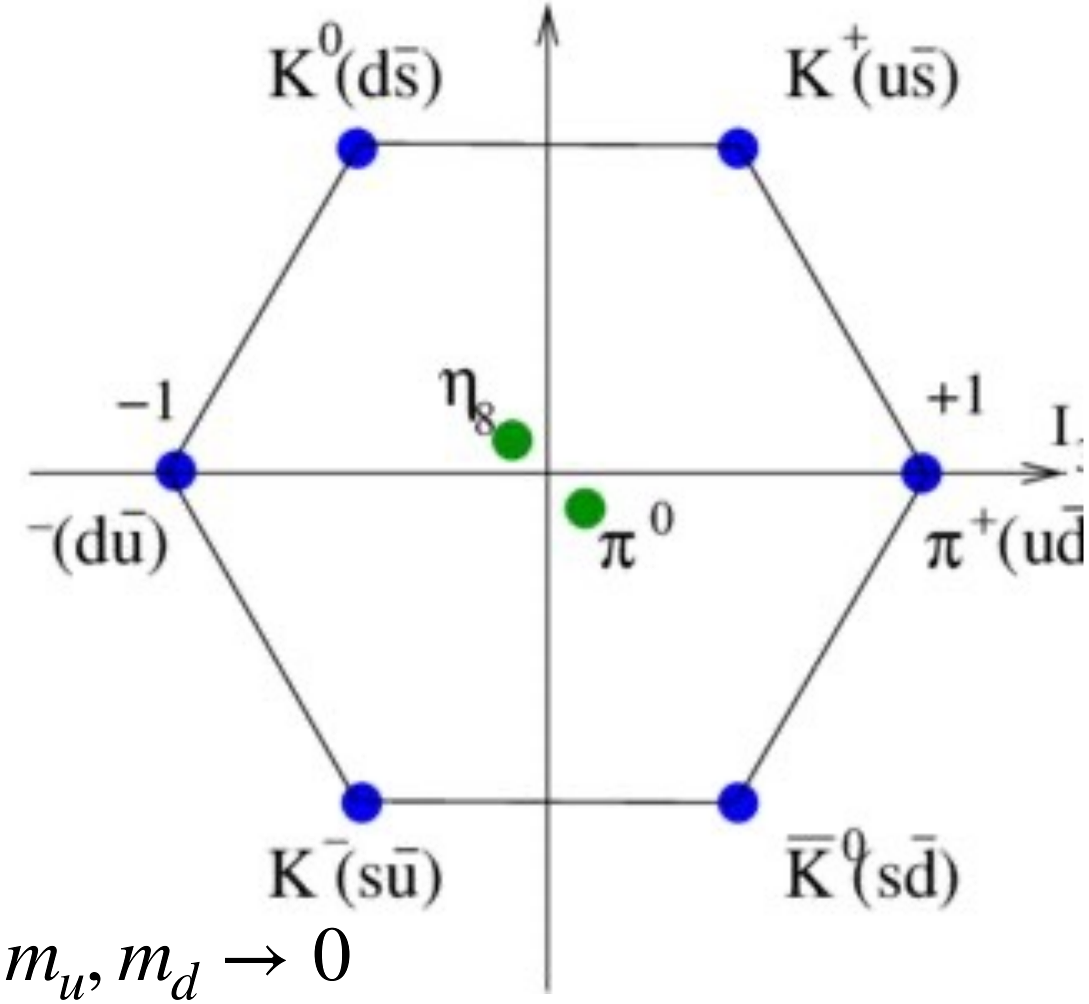
$SU(3)_L \times SU(3)_R$ chiral symmetry (group of dimension 8+8)

is spontaneously broken to $SU(3)$ flavor symmetry (group of dimension 8)

$\Longrightarrow$ (8+8) - 8 = 8 light Goldstone bosons

masses would be 0 if quark masses were 0

Quantum ChromoDynamics



Low-energy spectrum of QCD

Pseudoscalar mesons $J^P = 0^-$

pions: π^-, π^0, π^+	mass = 138 MeV
kaons: $K^+, K^-, K^0, \bar{K}^0$	496 MeV
eta: η	548 MeV
η'	958 MeV

pions: $SU(2)$ isospin triplet, masses go to 0 as $m_u, m_d \to 0$

pions, kaons, η: $SU(3)$ flavor octet, masses go to 0 as $m_u, m_d, m_s \to 0$

Vector mesons $J^P = 1^-$

ρ^-, ρ^0, ρ^+	mass = 770 MeV
ω	782 MeV
$K^{*+}, K^{*-}, K^{*0}, \bar{K}^{*0}$	892 MeV
ϕ	1020 MeV

Mesons with other J^P

masses significantly larger

(except $J^P = 0^+$: mass of $f_0(500) = \sigma \approx 500$ MeV)

Effective Field Theory

Stephen Weinberg:

QFT should describe particles only up to some high energy scale Λ
(set by masses of lightest particles not described explicitly)

- systematically improvable accuracy
  by expanding to higher order in energy/Λ
- infinitely many parameters but finite number at any given order in the energy expansion
- most general theory consistent with unitarity,
  analyticity,
  cluster decomposition,
  and assumed symmetries.
  AND indistinguishability of
  identical bosons or fermions

High-energy EFT

model-independent framework for higher energy physics

Low-energy EFT

systematic approximations for lower energy physics

Low-energy EFT for QCD

- identify high-energy scale Λ

  hadrons with masses > Λ will not be described explicitly in EFT
  (but their effects will be taken into account through parameters of EFT)

- identify lightest hadrons with masses smaller than Λ

  hadrons with masses < Λ will be described explicitly in EFT

- introduce fields that annihilate/create lightest hadrons

- identify action of symmetries of QCD on fields

  exact symmetries of QCD

  approximate symmetries of QCD

- construct most general Lagrangian for fields
  consistent with symmetries of QCD

- establish power counting for operators in the Lagrangian

  organizes EFT into expansion in powers of $1/\Lambda$

- determine parameters in the Lagrangian

  using experimental measurements of masses and reaction rates
  or calculate using lattice QCD in terms of α_s and quark masses

Low-energy EFT for QCD

- identify high-energy scale Λ

$SU(2)$ chiral EFT

Λ = mass of lightest meson whose mass remains nonzero in the limit $m_u, m_d \to 0$

$\Lambda \approx M_K = 496 \text{ MeV}$

$SU(3)$ chiral EFT

Λ = mass of lightest meson whose mass remains nonzero in the limit $m_u, m_d, m_s \to 0$

$\Lambda \approx M_\rho = 770 \text{ MeV}$ (what about σ ?)

- identify hadrons with masses smaller than Λ

$SU(2)$ chiral EFT

π^-, π^0, π^+: $SU(2)$ isospin triplet
mass 138 MeV

$SU(3)$ chiral EFT

$\pi^-, \pi^0, \pi^+, K^+, K^-, K^0, \bar{K}^0, \eta$: $SU(3)$ flavor octet
masses from 138 to 548 MeV

Low-energy EFT for QCD

- introduce light fields that annihilate/create lightest hadrons

<u>$SU(2)$ chiral EFT</u>

isospin triplet: π^-, π^0, π^+

real field $\pi^0(x)$

complex fields $\pi^+(x)$, $\pi^-(x) = \pi^+(x)^\dagger$

arrange into 2×2 hermitian matrix: $\xi(x) = \frac{1}{2} \begin{pmatrix} \pi_0(x) & \sqrt{2}\pi^+(x) \\ \sqrt{2}\pi^-(x) & -\pi^0(x) \end{pmatrix}$

<u>$SU(3)$ chiral EFT</u>

flavor octet: $\pi^-, \pi^0, \pi^+, K^+, K^-, K^0, \bar{K}^0, \eta$

8 real fields: $\xi^a(x)$, $a = 1,\ldots,8$

arrange into 3×3 hermitian matrix: $\xi(x) = \sum_{a=1}^{8} \xi^a(x) T^a$

8 generators of $SU(3)$: $T^a \quad [T^a, T^b] = i f^{abc}\, T^c$, $\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$

- identify action of symmetries of QCD on light fields

Low-energy EFT for QCD

- introduce fields that annihilate/create lightest hadrons

$SU(2)$ chiral EFT 2×2 hermitian matrix: $\xi(x) = \frac{1}{2} \begin{pmatrix} \pi_0(x) & \sqrt{2}\pi^+(x) \\ \sqrt{2}\pi^-(x) & -\pi^0(x) \end{pmatrix}$

$SU(3)$ chiral EFT 3×3 hermitian matrix: $\xi(x) = \sum_{a=1}^{8} \xi^a(x) T^a$

- identify action of symmetries of QCD on fields

exact symmetries of QCD: Lorentz invariance
Parity: pseudoscalar mesons have $P = -$
Charge conjugation: π^0, η have $C = +$

approximate $SU(N)$ flavor symmetry: $\xi(x) \longrightarrow U_V \, \xi(x) \, U_V^\dagger$

approximate $SU(N)_L \times SU(N)_R$ chiral symmetry: $\xi(x) \longrightarrow$??

Low-energy EFT for QCD

- identify action of symmetries of QCD on fields

exact symmetries of QCD: Lorentz invariance, Parity, Charge conjugation

approximate $SU(N)$ flavor symmetry: $\xi(x) \longrightarrow U_V\, \xi(x)\, U_V^\dagger$

approximate $SU(N)_L \times SU(N)_R$ chiral symmetry: ??
exploit spontaneous breaking to $SU(N)$ flavor symmetry !!

QCD vacuum state

condensation of quark-antiquark pairs with opposite helicities: $u_L\bar{u}_L,\, d_L\bar{d}_L,\, s_L\bar{s}_L$
$u_R\bar{u}_R,\, d_R\bar{d}_R,\, s_R\bar{s}_R$

small quark masses bias q and $\bar{q}$ to have same flavor

expectation values in QCD vacuum state: $\langle \bar{u}_R u_L \rangle = \langle \bar{d}_R d_L \rangle = \Delta \approx \langle \bar{s}_R s_L \rangle$

low-energy field configurations required by chiral symmetry

$SU(3)_L \times SU(3)_R$ chiral symmetry implies it costs very little energy
for pairing of u_L with linear superposition of $\bar{u}_L$, $\bar{d}_L$, and $\bar{s}_L$
for pairing of u_R with linear superposition of $\bar{u}_R$, $\bar{d}_R$, and $\bar{s}_R$

Low-energy EFT for QCD

- identify action of approximate $SU(N)_L \times SU(N)_R$ chiral symmetry

low-energy field configurations required by $SU(2)_L \times SU(2)_R$ chiral symmetry

condensation of quark-antiquark pairs with opposite helicities: $q_L\bar{q}_L,\, q_R\bar{q}_R$

pairing of $u_L,\, d_L$ with unitary superpositions of $\bar{u}_L,\, \bar{d}_L$

pairing of $u_R,\, d_R$ with unitary superpositions of $\bar{u}_R,\, \bar{d}_R$

matrix of expectation values: $\begin{pmatrix} \langle \bar{u}_R(x) u_L(x) \rangle & \langle \bar{d}_R(x) u_L(x) \rangle \\ \langle \bar{u}_R(x) d_L(x) \rangle & \langle \bar{d}_R(x) d_L(x) \rangle \end{pmatrix} = U(x)\,\Delta$

determines chiral field $U(x)$ with values on $SU(2)$ group manifold

low-energy field configurations required by $SU(3)_L \times SU(3)_R$ chiral symmetry

pairing of $u_L,\, d_L,\, s_L$ with unitary superpositions of $\bar{u}_L,\, \bar{d}_L$, and $\bar{s}_L$

pairing of $u_R,\, d_R,\, s_R$ with unitary superpositions of $\bar{u}_R,\, \bar{d}_R$, and $\bar{s}_R$

3×3 matrix of expectation values $\langle \bar{q}_R(x) q'_L(x) \rangle$

determines chiral field $U(x)$ with values on $SU(3)$ group manifold

Low-energy EFT for QCD

- identify action of approximate $SU(N)_L \times SU(N)_R$ chiral symmetry

low-energy field configurations required by $SU(2)_L \times SU(2)_R$ chiral symmetry

matrix of expectation values: $$\begin{pmatrix} \langle \bar{u}_R(x) u_L(x) \rangle & \langle \bar{d}_R(x) u_L(x) \rangle \\ \langle \bar{u}_R(x) d_L(x) \rangle & \langle \bar{d}_R(x) d_L(x) \rangle \end{pmatrix} = U(x)\,\Delta$$

chiral field $U(x)$ has values on $SU(2)$ group manifold

exponential parametrization: $U(x) = \exp\big(2i\,\xi(x)/F_\pi\big)$, $F_\pi = 93\ \mathrm{MeV}$

$\xi(x)$ is 2×2 hermitian matrix of π fields

low-energy field configurations required by $SU(3)_L \times SU(3)_R$ chiral symmetry

chiral field $U(x)$ has values on $SU(3)$ group manifold

exponential parametrization: $U(x) = \exp\big(2i\,\xi(x)/F_\pi\big)$,

$\xi(x)$ is 3×3 hermitian matrix of π, K, η fields

$SU(N)$ flavor symmetry: $U(x) \longrightarrow U_V\, U(x)\, U_V^\dagger$

$\xi(x) \longrightarrow U_V\, \xi(x)\, U_V^\dagger$

$SU(N)_L \times SU(N)_R$ chiral symmetry: $U(x) \longrightarrow U_L\, U(x)\, U_R^\dagger$

$\xi(x) \longrightarrow$ complicated nonlinear function of $\xi(x)$, U_L, U_R

Chiral EFT for QCD (massless quarks)

$SU(N)_L \times SU(N)_R$ chiral symmetry

spontaneously broken to $SU(N)$ flavor symmetry

$\Longrightarrow$ low-energy field configurations described by chiral field $U(x)$

with values on $SU(N)$ group manifold

Symmetries
- Lorentz invariance
- Parity
- Charge conjugation: $U(x) \longrightarrow U(x)^\dagger$
- exact $SU(N)$ flavor symmetry:

$$U(x) \longrightarrow U_V\, U(x)\, U_V^\dagger, \quad U_V \in SU(N)$$

- exact $SU(N)_L \times SU(N)_R$ chiral symmetry:

$$U(x) \longrightarrow U_L\, U(x)\, U_R^\dagger, \quad U_L, U_R \in SU(N)$$

Construct most general Lagrangian for $U(x)$ and its derivatives

consistent with the symmetries

High-energy scale: Λ

systematically improvable by expanding in energy/Λ

Chiral EFT for QCD (massless quarks)

Constraints on terms in Lagrangian

- Lorentz invariance $\Longrightarrow$ balanced Lorentz indices
- Parity $\Longrightarrow$ no Levi-Civita symbols $\epsilon_{\mu\nu\lambda\sigma}$
- Charge conjugation $\Longrightarrow$ invariant under $U \leftrightarrow U^\dagger$
- $SU(N)$ flavor symmetry: implied by $SU(N)_L \times SU(N)_R$ chiral symmetry
- $SU(N)_L \times SU(N)_R$ chiral symmetry: $U(x) \longrightarrow U_L\, U(x)\, U_R^\dagger$

$$U(x)^\dagger \longrightarrow U_R\, U(x)^\dagger\, U_L^\dagger$$

invariant $\Longrightarrow$ trace of products of (derivatives of) $U(x)$ and (derivatives of) $U(x)^\dagger$ with alternating $U(x)$ and $U(x)^\dagger$

2-derivative operator: $\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]$

4-derivative operators: $\left(\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]\right)^2$

$$\mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\nu U\right] \mathrm{Tr}\left[\partial^\mu U^\dagger \partial^\nu U\right]$$

$$\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right]$$ (redundant if *N*=2)

6-derivative operators: 4 for *N*=3 (3 for *N*=2)

Chiral EFT for QCD (massless quarks)

chiral field $U(x)$ with values on $SU(N)$ group manifold

Construct most general Lagrangian consistent with symmetries

- Lorentz invariance
- Parity
- Charge conjugation: $U(x) \longrightarrow U(x)^\dagger$
- exact $SU(N)$ flavor symmetry: $U(x) \longrightarrow U_V\, U(x)\, U_V^\dagger, \quad U_V \in SU(3)$
- exact $SU(N)_L \times SU(N)_R$ chiral symmetry: $U(x) \longrightarrow U_L\, U(x)\, U_R^\dagger, \quad U_L, U_R \in SU(3)$

Scaling dimensions

chiral field $U(x)$: $\quad d = 0$

derivative ∂_μ: $\quad d = 1$

Effective Lagrangian

$$\mathscr{L}_{\chi EFT} = \frac{1}{4} F_\pi^2 \, \mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \sum_{d \geq 4} \frac{1}{\Lambda^{4-d}} \sum_n c_{d,n} \mathscr{O}_{d,n}$$

- sum over scaling dimensions: $d = 4,6,8,\ldots$
- for each d, sum over independent operators with scaling dimensions d

Chiral EFT for QCD (massless quarks)

χEFT at Leading Order

$$\mathscr{L}_{\rm LO} = \tfrac{1}{4}F_\pi^2\,\mathrm{Tr}\big[\partial_\mu U^\dagger \partial^\mu U\big]$$

$F_\pi = 93\ \mathrm{MeV}$ “pion decay constant“

determine from decay rate for $\pi^+ \to \mu^+\nu_\mu$ in electroweak extension of χEFT

(or calculate using lattice QCD in terms of α_s and quark masses)

exponential parametrization of chiral field:

$$U(x) = \exp\left(\frac{2i}{F_\pi}\sum_a \xi^a(x) T^a\right)$$

real fields: $\xi^a(x),\ a = 1,\ldots,N^2-1$

generators of $SU(N)$: T^a satisfy $[T^a, T^b] = i f^{abc}\,T^c,\ \mathrm{Tr}[T^aT^b] = \frac{1}{2}\delta^{ab}$

Expand $\mathscr{L}_2$ in powers of ξ^a:

$$\mathscr{L}_2 = \frac{1}{2}\sum_a \partial^\mu\xi^a\partial_\mu\xi^a + \ldots$$

$\Longrightarrow N^2-1$ massless bosons (Goldstone bosons from chiral symmetry breaking)

4-boson, 6-boson, … interactions that go to 0 as 4-momentum $\to 0$ (Adler zeros)

Chiral EFT for QCD (massless quarks)

χEFT Lagrangian (for massless quarks)

$$\mathscr{L}_{\chi\mathrm{EFT}} = \frac{1}{4}F_\pi^2 \,\mathrm{Tr}\left[\partial^\mu U \partial_\mu U^\dagger\right] + \mathscr{L}_4 + \mathscr{L}_6 + \ldots$$

Next-to-Leading Order: scaling dimension 4

$$\mathscr{L}_4 = L_1\left(\mathrm{Tr}\left[\partial^\mu U \partial_\mu U^\dagger\right]\right)^2 + L_2 \mathrm{Tr}\left[\partial_\mu U \partial_\nu U^\dagger\right]\,\mathrm{Tr}\left[\partial^\mu U \partial^\nu U^\dagger\right] + L_3 \mathrm{Tr}\left[\partial^\mu U \partial_\mu U^\dagger \partial^\nu U \partial_\nu U^\dagger\right]$$

dimensionless coefficients: determine from experiment or calculate using lattice QCD

$SU(3)$ chiral EFT: 3 additional terms

$SU(2)$ chiral EFT: 2 additional terms (L_3 term is redundant)

Next-to-Next-to-Leading Order: scaling dimension 6

coefficients proportional to $1/\Lambda^2$: calculate using lattice QCD ?

$SU(3)$ chiral EFT: 4 additional terms

$SU(2)$ chiral EFT: 3 additional terms

If F_π is treated as order Λ,

low-energy expansion is a systematic expansion in $(\mathrm{energy}/\Lambda)^2$

Chiral EFT for QCD (nonzero quark masses)

$SU(N)_L \times SU(N)_R$ chiral symmetry

spontaneously broken to $SU(N)$ flavor symmetry

and also explicitly broken by light quark masses

$\Longrightarrow$ low-energy field configurations described by chiral field $U(x)$

with values on $SU(N)$ group manifold

Symmetries
- Lorentz invariance
- Parity
- Charge conjugation: $U(x) \longrightarrow U(x)^\dagger$
- approximate $SU(N)$ flavor symmetry:

  $$U(x) \longrightarrow U_V\, U(x)\, U_V^\dagger, \quad U_V \in SU(N)$$
- approximate $SU(N)_L \times SU(N)_R$ chiral symmetry:

  $$U(x) \longrightarrow U_L\, U(x)\, U_R^\dagger, \quad U_L, U_R \in SU(N)$$

Construct most general Lagrangian for $U(x)$ and its derivatives

consistent with the exact symmetries

and the approximate symmetries

High-energy scale: Λ

systematically improvable by expanding in powers of $1/\Lambda$

Chiral EFT for QCD (nonzero quark massess)

In QCD,

$SU(2)_L \times SU(2)_R$ chiral symmetry is explicitly broken by *u, d* quark masses

$SU(3)_L \times SU(3)_R$ chiral symmetry is explicitly broken by *u*, *d*, *s* quark masses

mass term in Lagrangian:

$$\mathscr{L}_{\rm mass} = - m_u \bar{u} u - m_d \bar{d} d - m_s \bar{s} s$$

Column vectors of quark spinor fields: $Q_L(x) = \begin{pmatrix} u_L(x) \\ d_L(x) \\ s_l(x) \end{pmatrix}, \quad Q_R(x) = \begin{pmatrix} u_R(x) \\ d_R(x) \\ s_R(x) \end{pmatrix}$

$SU(3)_L \times SU(3)_R$ chiral symmetry: $Q_L(x) \longrightarrow U_L\, Q_L(x), \;\; Q_R(x) \longrightarrow U_L\, Q_R(x)$

Quark mass matrix: $M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

$$\mathscr{L}_{\rm mass} = - \bar{Q}_L M Q_R - \bar{Q}_R M^\dagger Q_L$$

If M is replaced by general 3×3 complex matrix of background fields

then $\mathscr{L}_{\rm mass}$ is invariant under $SU(3)_L \times SU(3)_R$ chiral symmetry

if M transforms as $M \longrightarrow U_L\, M\, U_R^\dagger$

Chiral EFT for QCD (nonzero quark massess)

In chiral EFT,

to break $SU(N)_L \times SU(N)_R$ chiral symmetry in the same way as in QCD

construct EFT for the chiral field $U(x)$

in the presence of a mass matrix M of background fields

that respects the symmetry: $U(x) \longrightarrow U_L\, U(x)^\dagger\, U_R^\dagger$

$M \longrightarrow U_L\, M\, U_R^\dagger$

then set $M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

Mass terms

Leading Order: $M\partial^0$

$$\mathscr{L}_{\text{mass,LO}} = 2\,B\,\mathrm{Tr}\left[M^\dagger\, U + U^\dagger\, M\right]$$

Next-to-Leading Order: $M\partial^2$ and $M^2\partial^0$

$$\begin{aligned}\mathscr{L}_{\text{mass,NLO}} = {} & L_4\,\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]\,\mathrm{Tr}\left[M^\dagger U + U^\dagger M\right] + L_5\,\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\left(M^\dagger U + U^\dagger M\right)\right] \\ & + L_6\left(\mathrm{Tr}\left[M^\dagger U + U^\dagger M\right]\right)^2 + L_7\left(\mathrm{Tr}\left[M^\dagger U - U^\dagger M\right]\right)^2 \\ & + L_8\,\mathrm{Tr}\left[M^\dagger U\, M^\dagger U + U^\dagger M U^\dagger M)\right]\end{aligned}$$

1 parameter at LO (determine from pseudoscalar meson masses)

5 parameters at NLO (calculate using lattice QCD ?)

Chiral EFT for QCD (nonzero quark massess)

mass matrix $M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

exponential parametrization of chiral field: $U(x) = \exp\left(\frac{2i}{F_\pi}\sum_a \xi^a(x) T^a\right)$

Mass term at Leading Order

$$\mathscr{L}_{\text{mass,LO}} = 2\,B\,\mathrm{Tr}\left[M^\dagger U + U^\dagger M^\dagger\right]$$

Expand $U(x)$ in powers of $\xi(x) = \sum_a \xi^a(x) T^a$:

$$\mathscr{L}_{\text{mass,LO}} = 4\,B\,\mathrm{Tr}[M] - \frac{16B}{F_\pi^2}\mathrm{Tr}\left[M\xi^2\right] + \ldots$$
$$= 4\,B\,(m_u + m_d + m_s) - \frac{16B}{F_\pi^2}\sum_a \mathrm{Tr}\left[MT^aT^a\right](\xi^a)^2 + \ldots$$

pseudoscalar meson masses: squares are linear in quark masses

$$m_\pi^2 = (8B/F_\pi^2)(m_u + m_d)$$
$$m_K^2 = (8B/F_\pi^2)(m_u + m_d + 2m_s)/2$$
$$m_\eta^2 = (8B/F_\pi^2)(m_u + m_d + 4m_s)/3$$

determine B from measured masses

<u>Chiral EFT for QCD</u> (nonzero quark massess)

<u>χEFT Lagrangian</u> (with nonzero quark masses)

$\mathscr{L}_{\chi\text{EFT}}$ is a function of chiral field $U(x)$ and its derivatives

Scaling dimension *d* is determined by numbers of derivatives ∂_μ

and numbers of quark mass matrices M

Term with schematic form $M^p \partial^q$ has scaling dimension *d = 2p + q*

χEFT at Leading order: scaling dimension 2

$$\mathscr{L}_{\chi\text{EFT,LO}} = \frac{1}{4} F_\pi^2 \,\text{Tr}\left[\partial^\mu U \partial_\mu U^\dagger\right] + 2\,B\,\text{Tr}\left[M^\dagger U + U^\dagger M\right]$$

1 terms with schematic form $M^0 \partial^2$

1 terms with schematic form $M \partial^0$

Next-to-Leading Order: scaling dimension 4

3 terms with schematic form $M^0 \partial^4$

2 terms with schematic form $M \partial^2$

3 terms with schematic form M^2

<u>If F_π is treated as order Λ,</u>

EFT expansion is a systematic expansion in $(\text{energy}/\Lambda)^2$

and $(\text{quark mass}/\Lambda)$ or $(\text{pseudoscalar mass}/\Lambda)^2$