Exercises for Chiral Perturbation Theory

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The exercises listed below are for all four lectures and some build upon each other. They are also mentioned in the lecture slides

1. $\pi^1\pi^1 \to \pi^2\pi^2$
   do this for the three different cases given in the lectures
   (a) Linear sigma model
   (b) Nonlinear sigma model
   (c) Lowest order ChPT
2. $\pi^1(p)\tilde{\sigma}(q) \to \pi^1(p')\tilde{\sigma}(q')$
   (a) Calculate it in the linear sigma model and show the amplitude vanishes for $p \to 0$
   (b) (after baryon ChPT) prove that the ampltide for small p, p' is reproduced by $\mathcal{L} = \tilde{\sigma}_v(v \cdot \partial) + c\tilde{\sigma}\langle u_\mu u^\mu \rangle$ and $q = m_\sigma v$
3. Check the CCWZ relations on slide 32 for the transformation of covariant derivatives
4. Derive the Gell-Mann–Oakes-Renner and Gell-Mann–Okuba relations in lowest order ChPT (slide 51 in the lectures)
5. (takes more time) Derive the value of F_0 from lowest order ChPT by comparing with the lifetime of the pion and calculating the decay $\pi^+\pi\mu^+\nu$ Use the values of input numbers as given in the particle data book (pdg.lbl.gov
6. Do the one-loop calculation for the pion mass (most of what you need is either on the slides or you have worked out in the first exercise)
7. (Can take a lot of time if you want to do it fully) Redo the steps in the nucleon mass to order p^3