Effective Field Theories for Heavy Quarks

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Plan

- Introduction and overview ($Q = c, b$, $q = u, d, s$)
- Heavy-light systems ($Q\bar{q}$), HQET
- Heavy-heavy systems ($Q\bar{Q}$), NRQCD
- Heavy-heavy systems ($Q\bar{Q}$), pNRQCD
- Exotic heavy-heavy systems ($Q\bar{Q}g$, $Q\bar{Q}q\bar{q}$, $QQ\bar{q}\bar{q}$, $\dots$), BOEFT

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- Heavy-light systems: $Q\bar{q}$ (also Qqq, $Qq\bar{q}\bar{q}$, $\ldots$)
- Heavy-heavy systems: $Q\bar{Q}$ (also $Q\bar{Q}g$, QQq, $Q\bar{Q}q\bar{q}$, $QQ\bar{q}\bar{q}$, $\ldots$)

QCD

QCD (early 70's) is the current theory of the strong interactions.

$$\mathcal{L}_{QCD} = \mathcal{L}_g + \sum_{q=u,d,s} \mathcal{L}_q + \sum_{Q=c,b,t} \mathcal{L}_Q$$

$$\mathcal{L}_g = -\frac{1}{2} tr\left(F_{\mu\nu}F^{\mu\nu}\right)$$

$$\mathcal{L}_q = \bar{q}\left(i\not{D} - m_q\right) q \quad , \quad \mathcal{L}_Q = \bar{Q}\left(i\not{D} - m_Q\right) Q$$

- $-igF_{\mu\nu} = [D_\mu, D_\nu]$, $D_\mu = \partial_\mu - igA_\mu$, $A_\mu = T^a A^a_\mu$,
- $A^a_\mu \in 8$ of $SU(3)$, the gluon fields
- q and $Q \in 3$ of $SU(3)$, the light and heavy quark fields
- T^a are the generators of 3 of $SU(3)$
- g is the QCD coupling constant

QCD

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Why do we distinguish heavy from light quarks?

$$m_u = 2.16^{+0.49}_{-0.26}\ \mathrm{MeV}\ ,\ m_d = 4.67^{+0.48}_{-0.17}\ \mathrm{MeV}\ ,\ m_s = 93.4^{+8.6}_{-3.4}\ \mathrm{MeV}$$

$$m_c = 1.27{\pm}0.02\ \mathrm{GeV}\ ,\ m_b = 4.18^{+0.03}_{-0.02}\ \mathrm{GeV}\ ,\ m_t = 173.69{\pm}0.30\ \mathrm{GeV}$$

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- There is also $\Lambda_{\mathrm{QCD}} \sim 300 - 700\,\mathrm{MeV}$, dynamically generated scale. Heavy quarks have $m_Q \gg \Lambda_{\mathrm{QCD}}$
- Asymptotic Freedom implies $\alpha_{\mathrm{s}}(m_Q) \ll 1$, $\alpha_{\mathrm{s}} = g^2/4\pi$

QCD

The light quarks fulfil $m_q \ll \Lambda_{\mathrm{QCD}} \Longrightarrow$ there is an approximate $U(3) \times U(3)$ chiral symmetry,

- It is explicitely broken in the quantum theory down to $U(1) \times SU(3) \times SU(3)$
- And spontaneously broken down to flavor $SU(3)$ producing pseudo-Goldstone bosons, $G = \{\pi, K\ , \eta\}$, $m_G \ll \Lambda_{\mathrm{QCD}}$

Lattice QCD

Put (Euclidean) QCD on a lattice of N^4 points separated a distance a

- a is an UV regulator, hence $1/a \gg m_h$, m_h being the typical scale of the process we wish to study (e.g. a hadron mass).
- Na is an IR cut-off, hence $1/Na \ll m_G$ due to the existence of Goldstone bosons, otherwise $1/Na \ll \Lambda_{\rm QCD}$ would be enough.

The second constrain holds for any system we wish to study. The first one changes depending on the system

Lattice QCD

- For hadrons involving light quarks only, we have $1/a \gg \Lambda_{\mathrm{QCD}}$
- For hadrons involving heavy quarks, we have $1/a \gg m_Q$

QCD lattice calculations involving heavy quarks are very costly. A wise way to proceed is to calculate (factorize) the effects at the scale m_Q using perturbation theory, and leave for a less costly lattice calculation the effects at lower scales.

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  - Exploit the hierarchy of energy scales
- The EFT gives equivalent physical results in the region where it holds
  - It may make apparent accidental symmetries in that region, which help constraining the physics.
  - Calculations are usually simpler.

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- Appelquist-Carazone decoupling theorem: the effects of a heavy particle at energies much smaller than its mass (m_Q) are suppressed by $1/m_Q$ (up to counterterms).
- More generally, the path integral formulation suggests that one can integrate out field modes in the regions where I need not calculate Green functions involving them.

E.g. I: the SM at low energies

For $p \ll m_{W,Z}$ the Standard Model (SM) reduces to

$$\mathcal{L}_{QCD+QED} = -\frac{1}{2} tr\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{4} F^{em}_{\mu\nu} F^{em\,\mu\nu} + \\ + \sum_{q=u,d,s} \bar{q}\left(i\not{D}^q - m_q\right) q + \sum_{Q=c,b} \bar{Q}\left(i\not{D}^Q - m_Q\right) Q + \\ + \sum_{l=e,\mu,\tau} \bar{l}\left(i\not{D}^l - m_l\right) l + \bar{\nu}_l i\not{\partial}\nu_l + \mathcal{O}(1/m_{W,Z})$$

- $-igF_{\mu\nu} - iee_q F^{em}_{\mu\nu} = [D^q_\mu, D^q_\nu]$, $D^{q,Q}_\mu = \partial_\mu - igA_\mu - iee_{q,Q}A^{em}_\mu$, $D^l_\mu = \partial_\mu - iee_l A^{em}_\mu$
- e, charge of the positron
- $e_{q,Q,l}$, the fraction of the charge of the electron that the corresponding particles have.

E.g. I: the SM at low energies

- $\mathcal{L}_{QCD+QED}$ is invariant under local $SU(3) \times U^{em}(1)$
- The weak interactions are power ($\mathcal{O}(1/m_{W,Z})$) suppresed
- $\mathcal{L}_{QCD+QED}$ is invariant under global $U(1)^{N_q+N_Q+2N_l}$. Flavor is an accidental symmetry of the SM at low energies.
- The top quark is not in the EFT: top quark physics requires the full SM as a starting point.
- Since $\alpha = e^2/4\pi \sim 1/137$ and $\alpha_s(m_Z) \sim 0.117$ the electromagnetic interactions are often (but not always) neglegible in front of the strong ones.

E.g. II: QCD at low energies

For $p \ll m_Q$ QCD reduces to

$$\mathcal{L}_{QCD} = \mathcal{L}_g + \sum_{q=u,d,s} \mathcal{L}_q + \mathcal{O}(1/m_Q^2)$$

- The possible gluonic operators appearing at $\mathcal{O}(1/m_Q^2)$ are

$$tr\left([D^\mu, F_{\mu\nu}][D_\rho, F^{\rho\nu}]\right) \;,\; tr\left([D^\mu, F_{\rho\nu}][D_\mu, F^{\rho\nu}]\right) \;,\; tr\left(F_{\rho\nu}F^{\nu\mu}F_\mu^\rho\right)$$

- Exercise 1 illustrates how the Appelquist-Carazone theorem works. You are supposed to calculate the coefficients of the two first operators at one loop, as well as the additional counterterm which absorbs the contribution that is not power suppressed.

Heavy-light systems

$Q\bar{q}$ bound state , $m_Q >> \Lambda_{\rm QCD}$, $\alpha_{\rm s}(m_Q) << 1$

- The heavy quark is essentially at rest in the rest frame of the heavy meson
- If the heavy meson moves with velocity v, $v^2 = 1$, the heavy quark will also move with velocity v up to fluctuations $\sim \Lambda_{\rm QCD}/m_Q$, namely its momentum can be written as $p = m_Q v + k$, $k \sim \Lambda_{\rm QCD} \ll m_Q$
- Naïvely, we can write $Q(x) = e^{-im_Q vx} p_+ h_v^{(Q)}(x)$, $p_\pm = (1 \pm \not{v})/2$ projects onto the particle subspace, and $h_v^{(Q)}(x)$ contains the momentum fluctuations $k \sim \Lambda_{\rm QCD}$

Heavy-light systems

If this is done for all heavy quarks in the heavy quark part of the QCD lagrangian

$$\sum_{Q=c,b} \mathcal{L}_Q \longrightarrow \sum_{Q=c,b} \mathcal{L}^{(Q)}_{HQET} = \sum_{Q=c,b} \bar{h}^{(Q)}_v iv.Dh^{(Q)}_v$$

- It enjoys an accidental $SU(2N_Q)$ spin-flavor symmetry
  - $m_{b\bar{q}} - m_b \sim m_{c\bar{q}} - m_c \quad , \quad m_{bqq} - m_b \sim m_{cqq} - m_c$
  - For each Q, we have spin doublets $(S, S+1)$. $m_B \sim m_{B^*}$, $m_D \sim m_{D^*}$, $m_{\Sigma_c} \sim m^*_{\Sigma_c}$, $m_{\Sigma_b} \sim m^*_{\Sigma_b}$
- One expects the naïve picture to be modified by
  - Power corrections: $\sim 1/m_Q$ suppressed terms
  - Radiative corrections: $\sim \alpha_{\rm s}(m_Q)$ suppressed terms

Heavy-heavy systems

$Q\bar{Q}$ bound state = heavy quarkonium , $m_Q >> \Lambda_{\rm QCD}$, $\alpha_{\rm s}(m_Q) << 1$

- The heavy quarks in the rest frame of the heavy quarkonium are not at rest anymore, but move slowly ${\rm v} << 1$.
  Achtung: ${\rm v}$ must not be mistaken with v, the velocity of the meson
- In the heavy quarkonium rest frame, the typical momentum of the heavy quarks $p = m_Q v + k$ ($v = (1, \mathbf{0})$) is such that $k^0 \sim m_Q {\rm v}^2$, $\mathbf{k} \sim m_Q {\rm v}$
- Note that the only difference with HQET is that in HQET $k^0 \sim \mathbf{k} \ll m_Q$, whereas here we have $k^0 \sim m_Q {\rm v}^2 \ll \mathbf{k} \sim m_Q {\rm v} \ll m_Q$.

Heavy-heavy systems

We face a multi-scale problem:

- $m_Q >> m_Q \mathrm{v} >> m_Q \mathrm{v}^2$
- $m_Q >> \Lambda_{\mathrm{QCD}}$

Naïve derivation of NRQCD: make $SU(3)$ gauge invariant the lagrangian of a non-relativistic particle,

$$\mathcal{L}_Q + \mathcal{L}_{\bar{Q}} \longrightarrow \mathcal{L}_{NRQCD}^{Q\bar{Q}} =$$

$$\psi^\dagger \left(iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 \right) \psi + \chi^\dagger \left(-iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 \right) \chi$$

$\psi \sim h_v$ in HQET, Pauli spinor field that annihilates non-relativistic quarks; $\chi \sim h_{-v}$ in HQET, Pauli spinor field that creates non-relativistic anti-quarks.

Heavy-heavy systems

$$\mathcal{L}_{NRQCD}^{Q\bar{Q}} = \psi^\dagger \left(iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 \right) \psi + \chi^\dagger \left(-iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 \right) \chi$$

- Note that the heavy flavor symmetry of HQET is lost, but the $SU(2)$ spin symmetry for each heavy flavor remains $\Longrightarrow$ $S = 0, 1$ states approximately degenerated, e.g.
  $m_{J/\psi} \sim m_{\eta_c}$, $m_\Upsilon \sim m_{\eta_b}$, $m_{B_c} \sim m_{B_c^*}$, $m_{\Xi_{cc}} \sim m_{\Xi_{cc}^*}$
- One expects the naïve picture to be modified by
  - Power corrections: $\sim 1/m_Q$ suppressed terms (including operators with four heavy fields)
  - Radiative corrections: $\sim \alpha_s(m_Q)$ suppressed terms

Heavy-heavy systems

- We face a multi-scale problem:
  - $m_Q >> m_Q\mathrm{v} >> m_Q\mathrm{v}^2$
  - $m_Q >> \Lambda_{\mathrm{QCD}}$
- pNRQCD disentangles the scales $m_Q\mathrm{v}$ and $m_Q\mathrm{v}^2$
  - If $m_Q\mathrm{v} \gg \Lambda_{\mathrm{QCD}}$ $\Longrightarrow$ Weak coupling regime:
    - Color singlet and color octet wave function fields
    - Untrasoft gluons $(E \sim m_Q\mathrm{v}^2)$ mediate transitions between them
    - The potentials can be calculated in perturbation theory of $\alpha_{\mathrm{s}}(m_Q\mathrm{v})$
    - Relevant for the ground states of $c\bar{c}$, $c\bar{b}$, $b\bar{b}$

Heavy-heavy systems

- We face a multi-scale problem:
  - $m_Q >> m_Q \mathrm{v} >> m_Q \mathrm{v}^2$
  - $m_Q >> \Lambda_{\mathrm{QCD}}$
- pNRQCD disentangles the scales $m_Q \mathrm{v}$ and $m_Q \mathrm{v}^2$
  - If $m_Q \mathrm{v} \sim \Lambda_{\mathrm{QCD}}$ $\Longrightarrow$ Strong coupling regime:
    - Color singlet wave function field
    - The potentials cannot be calculated in perturbation theory
    - Relevant for most of the $c\bar{c}$, $c\bar{b}$, $b\bar{b}$ states
  - The strong coupling regime can be generalized for exotic doubly heavy hadrons $\sim$ Born-Oppenheimer EFT (Ξ_c^{++}, T_{cc}^+, Z_c, Z_b, $\ldots$)

Heavy to light decays$(Q\bar{q})$

$Q\bar{q}$ (and the $b\bar{c}$, $c\bar{b}$) decay through the weak interactions

- Leptonic $Q\bar{q} \to l\bar{\nu}$, easy, given in terms of $f_{Q\bar{q}}$
- Semileptonic, $Q \to q' l\bar{\nu}$
  - For generic $p_{q'}$, $p_{q'}^2 \gg \Lambda_{\rm QCD}^2$, the inclusive width can be calculated using perturbative QCD + HQET
  - For $p_{q'}^2 \sim \Lambda_{\rm QCD}^2$, perturbative QCD cannot be used. There are two cases
    - All components are of the same order $p_{q'} \sim \Lambda_{\rm QCD}$, the decay is parameterized by a soft function.
    - $p_{q'}^0 \gg \Lambda_{\rm QCD}$ and $|\mathbf{p}_{q'}| \gg \Lambda_{\rm QCD}$, we have a jet-like event. SCET is designed to deal with this case.

Heavy to light decays($Q\bar{q}$)

$Q\bar{q}$ (and the $b\bar{c}$, $c\bar{b}$) decay through the weak interactions (Cont.)

- Non-leptonic $Q \to q'q''\bar{q}'''$
  - Fully inclusive, can be calculated using perturbative QCD + HQET
  - Exclusive two body, $\mathrm{SCET_{II}}$ is suitable for those

Heavy to light decays($Q\bar{Q}$)

$Q\bar{Q}$ (except for $b\bar{c}$, $c\bar{b}$) decay through the strong and electromagnetic interactions

- Electromagnetic decays, easy, given in terms of $f_{Q\bar{Q}} \sim$ wave function at the origin.
- Strong decays
  - Fully inclusive, can be calculated using perturbative QCD + NRQCD
  - Exclusive two body, SCET$_{\rm II}$ is suitable for those
- Radiative decays ($Q\bar{Q} \to X\gamma$)
  - For generic p_γ, perturbative QCD + NRQCD
  - For $p_\gamma^0 \sim m_Q/2$, we have jet-like events, SCET is suitable.