Heavy quarks, NR EFTs
Exercices Lectures 1,2 and 3

1. Calculate the gluon vacuum polarization due to a heavy quark for gluon momenta much smaller than the heavy quark mass at one loop in the $\overline{\rm MS}$ scheme:

1.1 Obtain the matching coefficients a, b and c of the effective theory that reproduces this result at $\mathcal{O}(1/m^2)$

$$a\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})+b\,\mathrm{tr}\left([D^\mu,F_{\mu\nu}][D_\rho,F^{\rho\nu}]\right)+c\,\mathrm{tr}\left([D^\mu,F_{\rho\nu}][D_\mu,F^{\rho\nu}]\right)+d\,\mathrm{tr}\left(F_{\rho\nu}F^{\nu\mu}F^{\rho}_{\mu}\right)\ .$$

1.2 Show that the last operator can be written in terms of the two ones in the middle.

1.3 Which terms can be eliminated using local field redefinitions?

1.4 Show that the (constant) local field redefinition that brings $(-1/2+a)\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$ to the standard normalization makes $\alpha_{\rm s}$ run with one less flavor.

2. Calculate the anomalous dimension of the chromomagnetic operator (c_F) in HQET at one loop in the $\overline{\rm MS}$ scheme.

3. Calculate the ultrasoft loop in the static limit of pNRQCD (i.e. neglecting all $1/m$ terms) and show that it cancels the scale dependence of α_{V_s}. Use DR and the $\overline{\rm MS}$ scheme.

4. Consider the following potential in pNRQCD $V(\mathbf{r})=d\,\delta(\mathbf{r})/m^2$, which can be treated as a perturbation:

4.1 Show that at second order in quantum mechanical perturbation theory it produces a logarithmic divergence (in DR, with a hard cut-off it also produces a linear one). Use the fact that in order to analyze the UV behavior the Coulomb potential can also be treated as a perturbation).

4.2 Identify the counterterm which absorbs the UV divergence, and calculate the residue of the $1/\epsilon$ pole.

4.3 What is the corresponding diagram for this calculation in NRQCD?