

Perturbative QCD at finite baryon density, Problem set  
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1. Using the cutting rules introduced in the lectures (see also 1609.04339), sketch the computation of the following massless Euclidean two-loop integral at vanishing temperature:

$$S = \Lambda^{4\epsilon} \int \frac{d^{4-2\epsilon} P}{(2\pi)^{4-2\epsilon}} \int \frac{d^{4-2\epsilon} Q}{(2\pi)^{4-2\epsilon}} \frac{1}{\tilde{P}^2 \tilde{Q}^2 (P - Q)^2}.$$

In this expression,  $P$  and  $Q$  are fermionic momenta at nonzero chemical potential, and the tildes remind us that their temporal components have been offset by  $i\mu$ .

Argue why the 0- and 1-cut contributions to the integral must vanish in dimensional regularization, and attempt an explicit analytic evaluation of the 2-cut part. If needed, some help can be obtained from appendix D.3 of 2304.05427.

2. Consider the  $T \rightarrow 0$  limit of the fermionic one-loop sum-integral

$$I = \not\int_{\{P\}} \frac{1}{\tilde{P}^4}$$

and evaluate it in two independent ways:

- a. Beginning from nonzero  $T$  and taking  $T \rightarrow 0$  by hand,
- b. Setting  $T = 0$  in the beginning and proceeding by Residue calculus.

Do your answers agree? If not, why?

If need be, you may consult Appendix B.1 of hep-ph/0305183 and p.2 of 2208.14479.