EFT Exercises

1. The Hilbert series for a complex scalar field ϕ is

$$
\begin{aligned}
H(\phi,\phi^*,\partial) &= 1+\phi^*\phi+(\phi^*\phi)^2 \\
&\quad +\big[(\phi^*\phi)^3+(\phi^*\phi)^2\partial^2\big] \\
&\quad +\big[(\phi^*\phi)^4+(\phi^*\phi)^3\partial^2+(\phi^*\phi)^2\partial^4\big]+\dots
\end{aligned}
$$

The coefficient of $(\phi^*\phi)^2\partial^2$ indicates that there is only one independent dimension-6 term with 4 fields and 2 derivatives after eliminating EOM and IBP redundancies.

(a) Write down the 5 independent dimension-6 terms with the schematic form $(\phi^*\phi)^2\partial^2$.

(b) Identify the 3 independent terms that are real valued.

(c) Use IBP to show that the term $(\phi^*\phi)\partial_\mu\phi^*\partial^\mu\phi$ is redundant.

(d) Use EOM to show that the term $(\phi^*\phi)(\partial^2\phi^*\phi+\phi^*\partial^2\phi)$ is redundant.

The Lagrangian through dimension 6 can be written

$$
\begin{aligned}
\mathcal{L} &= \partial_\mu\phi^*\partial^\mu\phi-m^2(\phi^*\phi)^2-\frac{1}{4}\lambda(\phi^*\phi)^4 \\
&\quad +\frac{c_{6,1}}{\Lambda^2}(\phi^*\phi)^3+\frac{c_{6,2}}{\Lambda^2}(\phi^*\phi)\partial_\mu\phi^*\partial^\mu\phi+\frac{c_{6,3}}{\Lambda^2}(\phi^*\phi)\big(\partial^2\phi^*\phi+\phi^*\partial^2\phi\big).
\end{aligned}
$$

(For simplicity, you can set $c_{6,1}=c_{6,2}=0$ for the following parts.)

(e) Calculate the variation $\delta\mathcal{L}$ to first order in the variation $\delta\phi$ (with $\delta\phi^*=0$).

(f) Determine the field redefinition $\phi\longrightarrow\phi+\Phi$ that would be required to cancel the $c_{6,3}$ term in the Lagrangian.

(g) Use the invariance of the path integral under field redefinitions to show that the effects of the $c_{6,3}$ term are equivalent to a dimension-8 term. (It can therefore be omitted from the dimension-6 terms in the Lagrangian.)