Exercises: Perturbative Gradient Flow

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Asymptotic expansion of flow-time integrals. Evaluate the integral

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{e^{-t[k^2+(k-q)^2]}}{k^2(k-q)^2} \tag{1}$$

in the limit $tq^2 \ll 1$ up to the first non-vanishing order in $q^2 t$.

Flow lines. The generating functional which implies the flow equation reads

$$\begin{aligned} Z[J_B, J_L] \sim \int \mathcal{D}L\mathcal{D}B \, \exp\Big[- \int \mathrm{d}^4 x \int_0^\infty \mathrm{d}t \, \Big(L^a_\mu(t,x)(\partial_t - \Box_x) B^a_\mu(t,x) \\ - J_{B,\mu}(t,x) B_\mu(t,x) - L_\mu(t,x) J_{L,\mu}(t,x) \Big) \Big] \,, \end{aligned} \tag{2}$$

where we only kept the quadratic terms in the fields in the exponent. Using the fact that the integral measure is invariant under the shifts

$$\begin{aligned} L^a_\mu(t,x) &\to L^a_\mu(t,x) + \int \mathrm{d}^4 y \int_0^\infty \mathrm{d}s \, J_{B,\mu}(s,y) P(s-t, y-x) \,, \\ B^a_\mu(t,x) &\to B^a_\mu(t,x) + \int \mathrm{d}^4 y \int_0^\infty \mathrm{d}s \, P(t-s, x-y) J_{L,\mu}(s,y) \,, \end{aligned} \tag{3}$$

with

$$(\partial_t - \Box_x) P(t-s, x-y) = \delta(t-s)\delta(x-y) \,, \tag{4}$$

show that the mixed propagator is

$$\langle 0 | T B^a_\mu(t,x) L^b_\nu(s,y) | 0 \rangle = \delta^{ab} \delta_{\mu\nu} \, P(t-s, x-y) \,. \tag{5}$$