Problems in lattice perturbation theory

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Lattice photon propagator, part 1

In order to define a photon propagator with the right number (two) of degrees of freedom, we have to fix a gauge to eliminate the longitudinal one. On the lattice, we usually choose some discretized form of a Lorentz gauge condition

$$\mathcal{G}_{x,s}(A_\mu, \chi) = \delta\left(\sum_{\mu=0}^{3} \partial_\mu^b A_{\mu,x+se_\mu} - \chi_x\right) , \quad (1)$$

where χ_x is an arbitrary scalar field, and ∂_μ^b is the backward partial derivative. This constraint $\delta(\mathcal{G}_{x,s}(A_\mu, \chi))$ can be exponentiated by averaging over a Gaussian-distributed χ field in both the numerator and in the denominator, thus introducing the arbitrary[1] **gauge-fixing parameter** ξ. Work out the corresponding gauge-fixed action.

Note: The gauge fixing is much more complicated for QCD due to the non-Abelian nature of the fields. The full Fadeev-Popov procedure is needed, whose complications are almost an exact copy (besides the finiteness of all expressions) of the situation in the continuum.

[1] Given that the parameter ξ is arbitrary, we can equivalently define the gauge-fixing condition in terms of the unrescaled connection $\mathcal{A}_\mu$ or in terms of the rescaled connection A_μ. Redefining the parameter ξ absorbs the difference.

DPG

- Derive the lattice photon propagator for the plaquette action

$$S_{\text{Plaq}} = \sum_{n\in\Lambda} \sum_{\mu\neq\nu} \frac{1 - \mathrm{Re}\ W_{\mu,\nu}[U](n)}{2e_0^2} \ , \tag{2}$$

and the gauge fixing action

$$S_{\text{GF}} = \frac{a^4}{2\xi} \sum_{n\in\Lambda} \left[\sum_{\mu=0}^{3} \partial_\mu^b A_\mu(n + s\hat{\mu}) \right]^2 \ . \tag{3}$$

Feynman rules for scalar QED

- Derive the Feynman rules of scalar QED (no $\frac{\lambda}{4}|\varphi|^4$ term) relevant for the self-energy of the scalar field.

- Derive the one or two-gluon vertices for Wilson fermions coupled to lattice QED photons.

Self-energy of Wilson fermions

- Draw all diagrams contributing to the one-loop self-energy of Wilson fermions coupled to lattice QED fields.
- Calculate the lattice degree of divergence for these diagrams.
- Isolate the most divergent contributions in these diagrams.

QED vacuum polarization due to Wilson fermions

- Draw all diagrams for the Wilson fermion contribution to the QED vacuum polarization.
- Calculate the lattice degree of divergence for these diagrams.
- Isolate the most divergent contributions in these diagrams.
- Transform the integrands to see that the Wilson fermion contribution to the vacuum polarization is free from power-law divergences. It is not necessary to solve integrals to see this.