



RUB

RUHR-UNIVERSITÄT BOCHUM

CONSTRAINING THE TWO-NUCLEON FORCE IN CHIRAL EFT FROM THREE-NUCLEON DATA



Observing the unobservable off-shell behavior?

Sven Heihoff, Evgeny Epelbaum
contact: sven.heihoff@rub.de

Off-shell Low-Energy Constants (off-shell LECs)

- 2N potential in chiral EFT in the order N³LO in the 1S_0 partial wave:

$$\langle ^1S_0, p' | V^{(4)} | ^1S_0, p \rangle = D_{1S_0} p^2 p'^2 + D_{1S_0}^{\text{off}} (p^2 - p'^2)^2 + \dots$$

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- Conclusion:** Fixing off-shell LECs is equivalent to fixing arbitrary transformation angles!

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- Technicalities:**
 - Iterating Faddeev equation takes a lot of time \rightarrow emulator based on RBF interpolation
 - Fitting off-shell LECs takes ~ 1 min instead of ~ 1 week, cost: on average 3% error
 - Creating a reliable database of experimental 3N data

Improvement of 3N data description

