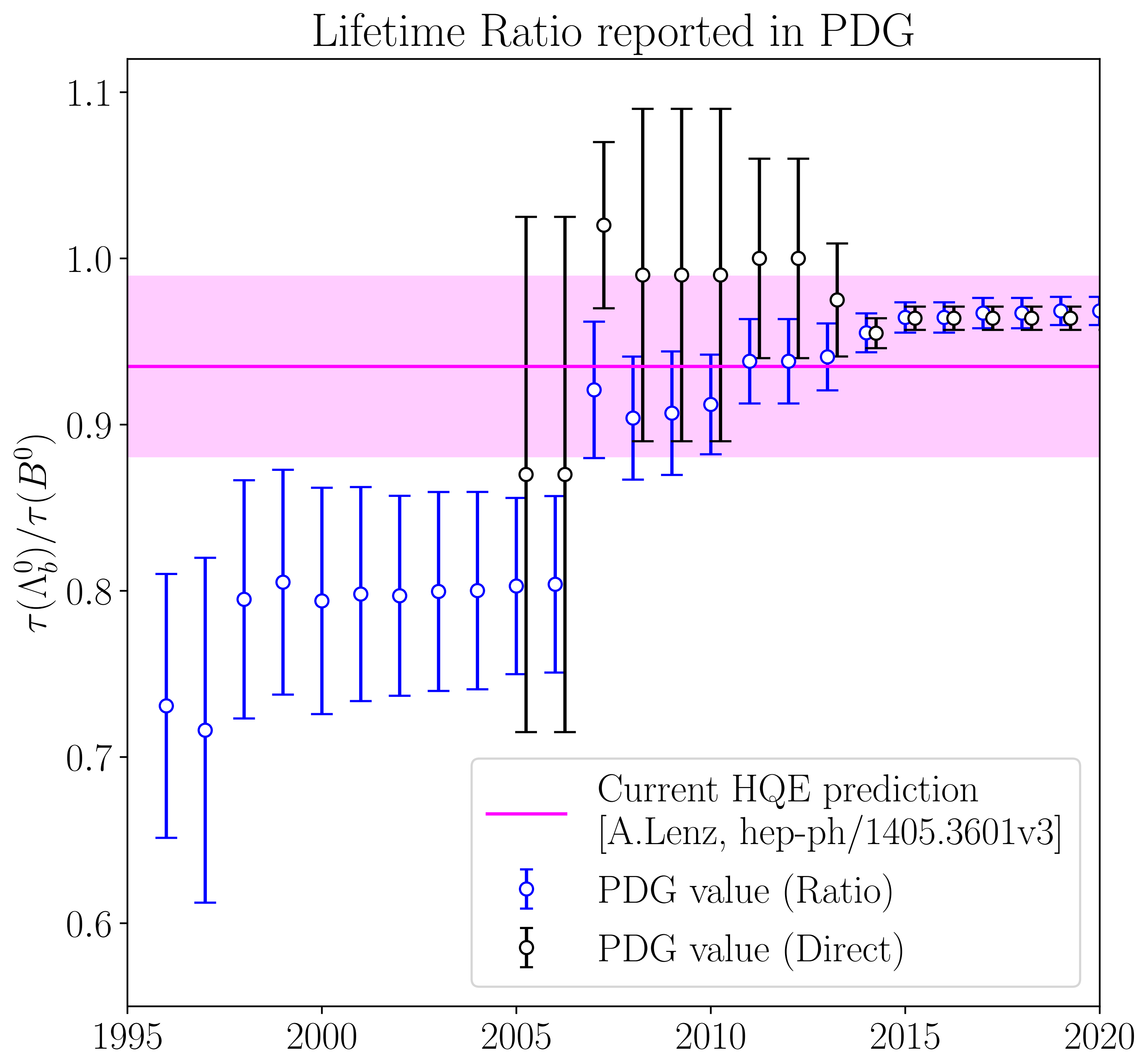
Inclusive Lifetimes of B-hadrons

joshlin@mit.edu

via Heavy Quark Expansion, [Review: A. Lenz, hep-ph/1405.3601]



- Large theoretical uncertainty on inclusive lifetimes of B-hadrons
- One contribution to uncertainty comes from 'Spectator Effects', matrix elements of these dimension-6 operators
- I'm trying to measure them on the lattice!

Four-Fermi theory of Weak Interactions

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}}\left[V_{cb}^* V_{us}(C_1 Q_1 + C_2 Q_2) + \cdots V_{cb}^* Q_l^c + \cdots\right]$$

$$Q_1 = (\bar{b}_i c_j)_{V-A}(\bar{u}_j s_i)_{V-A} \qquad Q_l^q = (\bar{b}_i q_i)_{V-A}(\bar{\nu}_l l)_{V-A}$$

↓

Optical Theorem

$$\Gamma(B \to X) = \frac{1}{2m_B}\langle B|\,\mathrm{Im}\, i\int d^4x T\left(H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0)\right)|B\rangle$$

↓

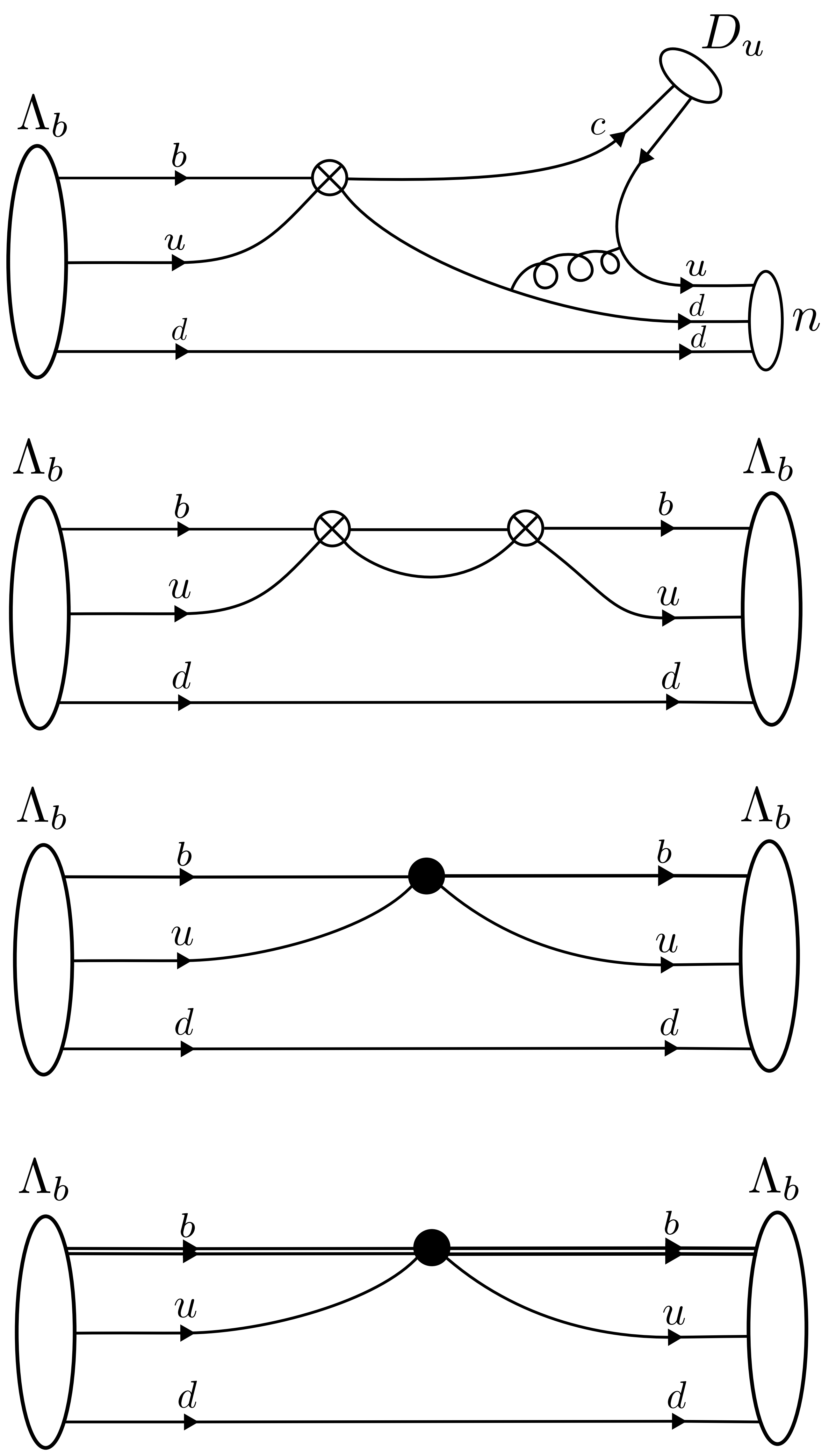
Operator Product Expansion

$$\hat{T} = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3}\left[c^{(3)}\bar{b}b + \frac{c^{(5)}}{m_b^2} g_s \bar{b}\sigma^{\mu\nu}G_{\mu\nu}b + \frac{1}{m_b^3}\sum_k c_k^{(6)} O_k^{(6)} + \cdots\right]$$

↓

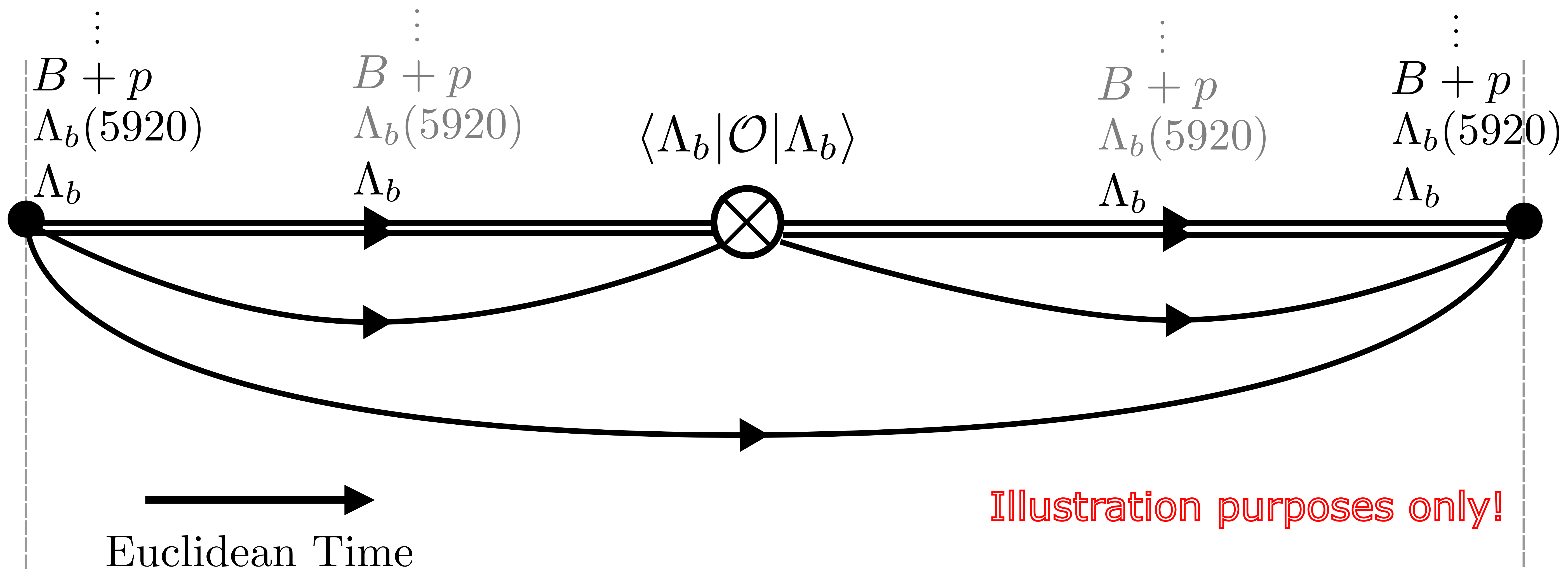
Heavy Quark Effective Theory

$$\bar{b}b = \overline{b_+}b_+ + \overline{b_+}\frac{D_T^2 + \frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}}{4m_b^2}b_+ + O\left(\frac{1}{m_b^3}\right)$$

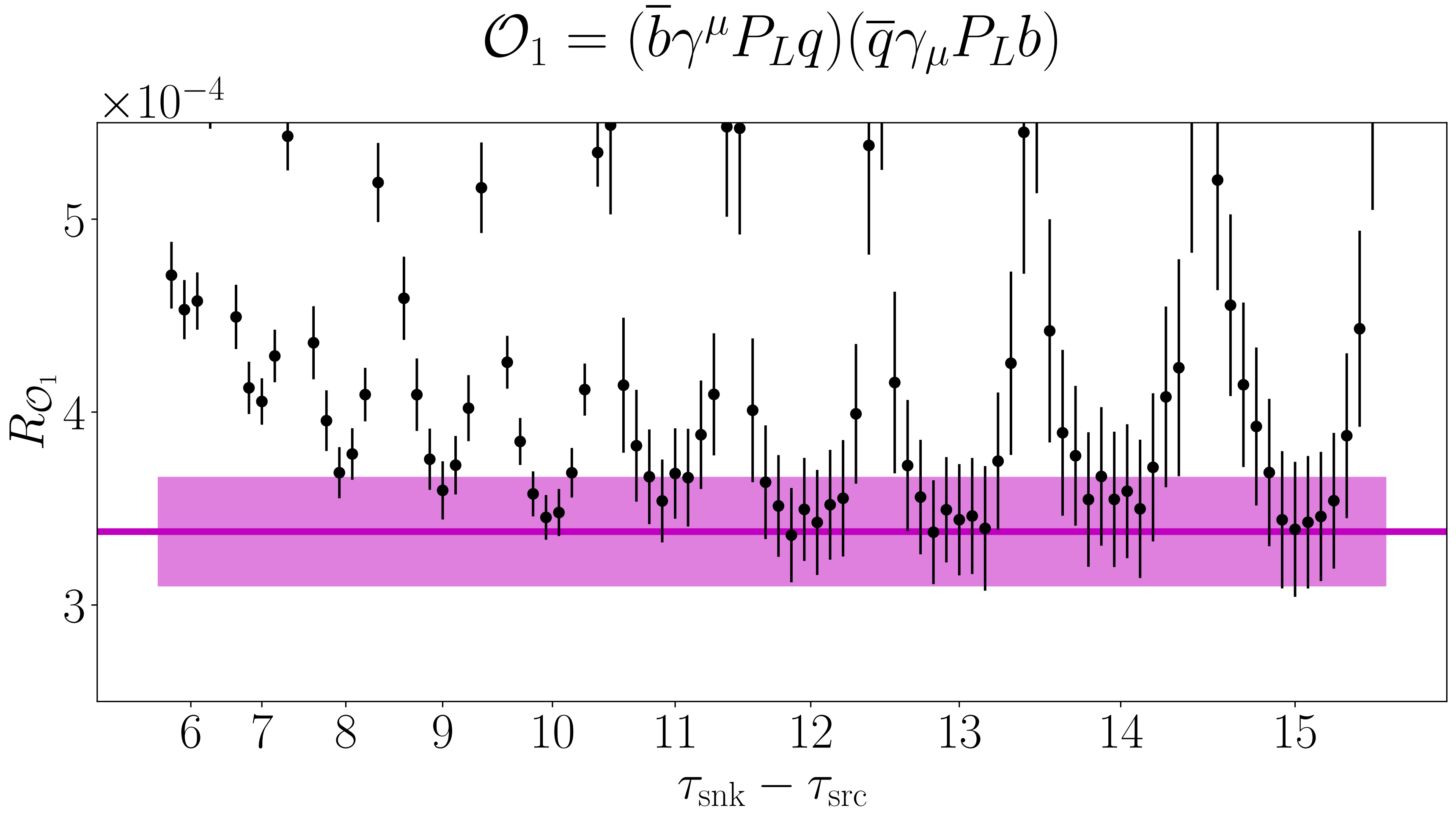


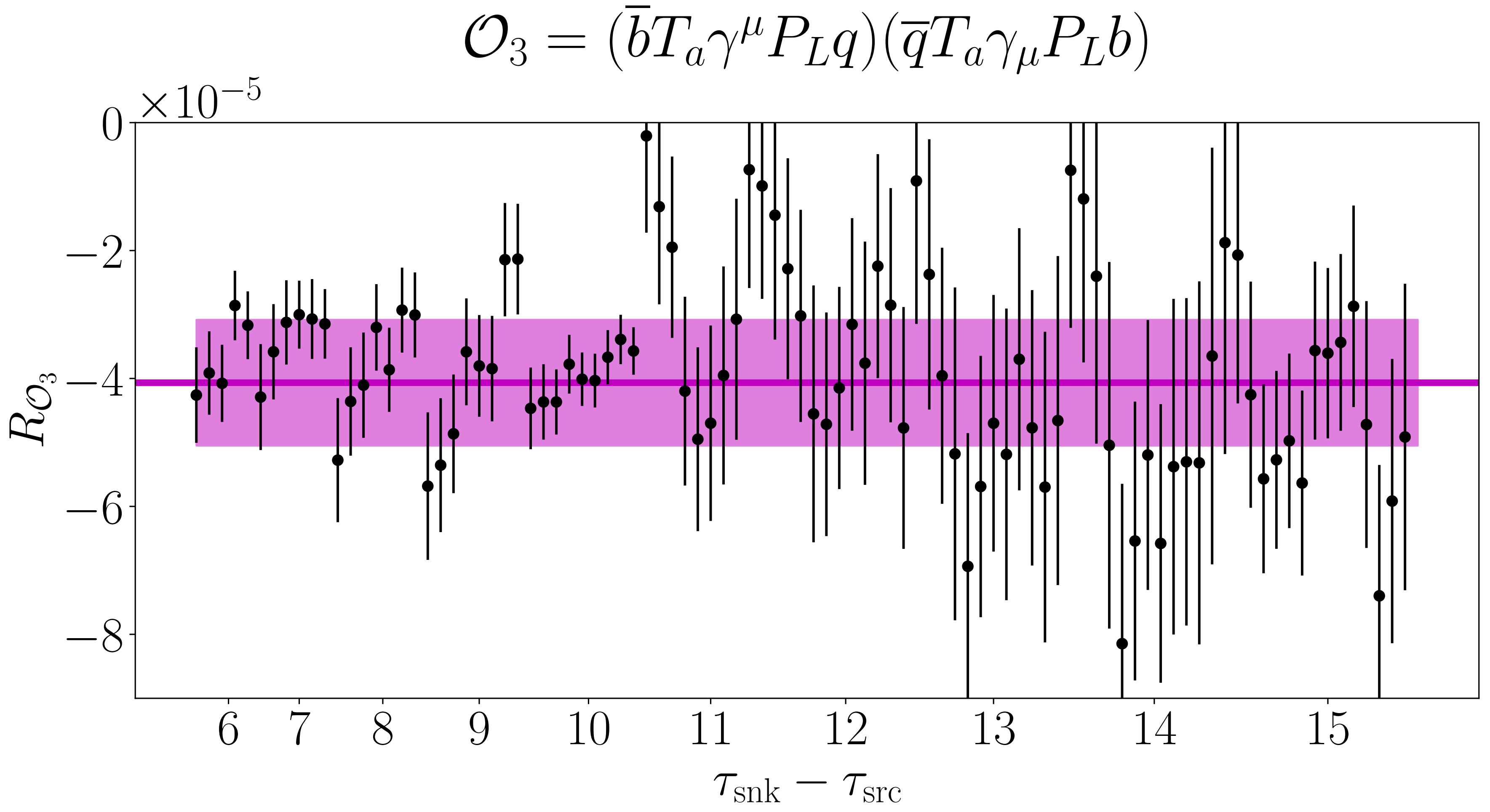
Matrix elements on the Lattice?

joshlin@mit.edu



- Three-point functions probe matrix elements!
- In practice, devise operators to have maximal overlap onto states of interest
- Use Ginsparg-Wilson fermions, in practice **Domain Wall Fermions** to prevent mixing with operators of different chiral representations.
- HQET propagators are static Wilson lines on the lattice, this is *noisy*. Use **Gradient Flow** to smear!





- Fitting is as complicated as you would like it to be!
- Coupled 2pt-3pt fits, bootstrap, varying fitting windows, Akaike information criterion, ...

(with W. Detmold and S. Meinel, arxiv/2212.09275)

Position-space Renormalization

joshlin@mit.edu

Bare Lattice Matrix Elements →

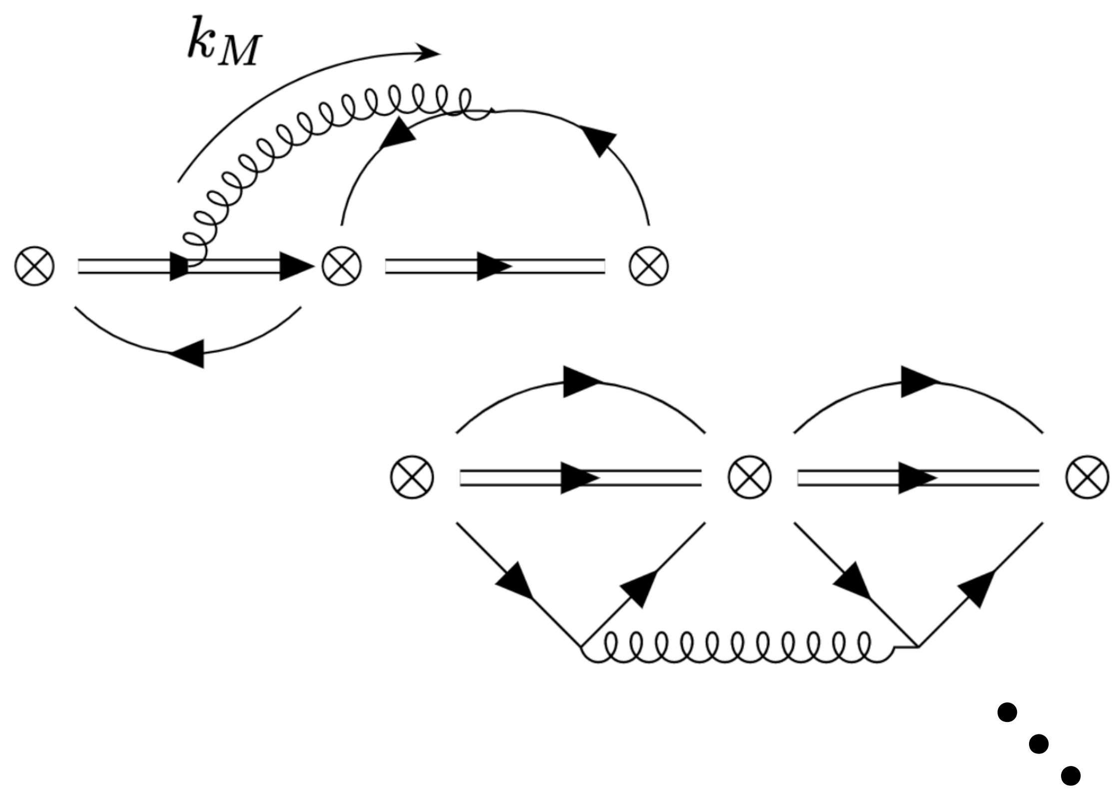
Lattice Perturbation Theory

- Gradient Flow
- RI-(s)MOM
- Shrodinger Functional
- Position-space scheme(!)

→ MS-renormalized matrix elements

- Gauge invariant (no Gribov copies)
- Similar window problem to other NP schemes
- Simple to implement on Lattice! Tradeoff is that you need to do position-space calculations. Extra scales in dim-reg integrals always make things more annoying!

Position-space scheme definition: $$\frac{\langle J_\alpha(x_0)\mathcal{O}_j^{(X)}(0)J_\alpha^\dagger(-x_0)\rangle}{\langle J_\alpha(x_0)J_\alpha^\dagger(-x_0)\rangle} = \frac{\langle J_\alpha(x_0)\mathcal{O}_j^{(0)}(0)J_\alpha^\dagger(-x_0)\rangle}{\langle J_\alpha(x_0)J_\alpha^\dagger(-x_0)\rangle}\Bigg|_{\text{Tree Value}}$$



→

$$\int \frac{d^dp_Ld^dp_Rd^dk}{(2\pi)^{3d}} \frac{e^{ip_Lx_L-ip_Rx_R}p_R^\alpha(p_R-k)^\beta(p_L-k)^\rho p_L^\delta}{(-p_L^2)(-(p_L-k)^2)(-(p_R-k)^2)(-p_R^2)(-k^2)}$$

$$\int \frac{d^dp_Ld^dp_Rd^dk}{(2\pi)^{3d}} \frac{p_R^\alpha(p_R-k)^\beta e^{ix_Lp_L}e^{-ix_Rp_R}}{(-p_R^2)(-(p_R-k)^2)(-k^2)(v\cdot(p_L-k))(v\cdot p_L)}$$

$\ddots$

→ **Integration by Parts** to get Master Integrals (do by hand/consult the manual)

Thanks!