

Static Force from the **Lattice** with **Gradient flow**

Julian Mayer-Steuerte*, Nora Brambilla, Viljami Leino, Antonio Vairo



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- We measure the static force on the lattice directly

- Instead of derivative of the static potential:

Wilson loop with E -field insertion

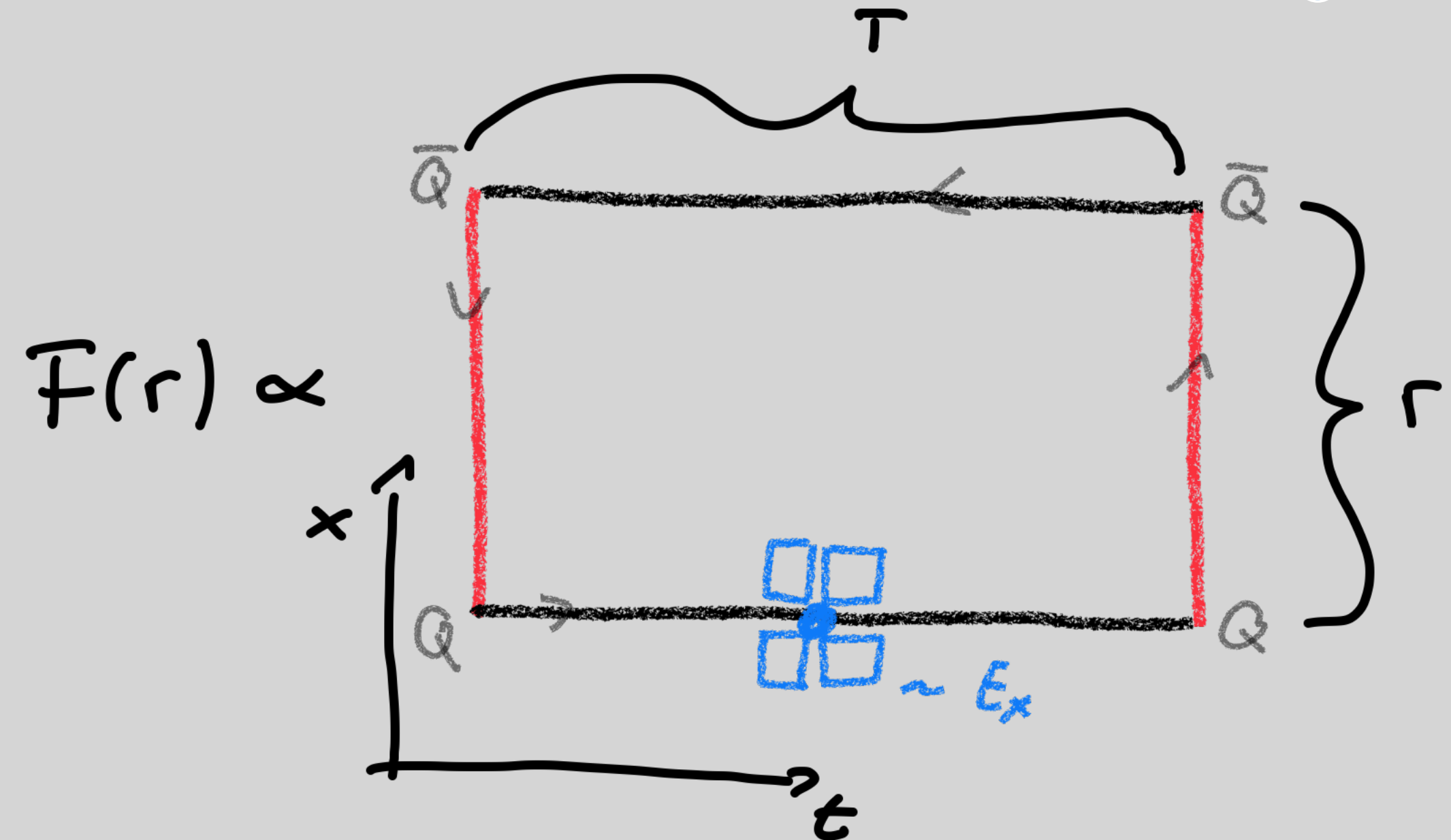
$$F(r) = \lim_{T \rightarrow \infty} \frac{\langle WE \rangle_{r,T}}{\langle W \rangle_{r,T}}$$

- Motivation:

- Comparable with $\partial_r V(r)$
- Preparation for similar objects required for NREFTs
- Standard Model parameter estimation, e.g. Λ_0

- Problems:

- Bad signal-to-noise ratio
- Discretized E -field discretization requires lattice only renormalization: $Z_E F(r)$



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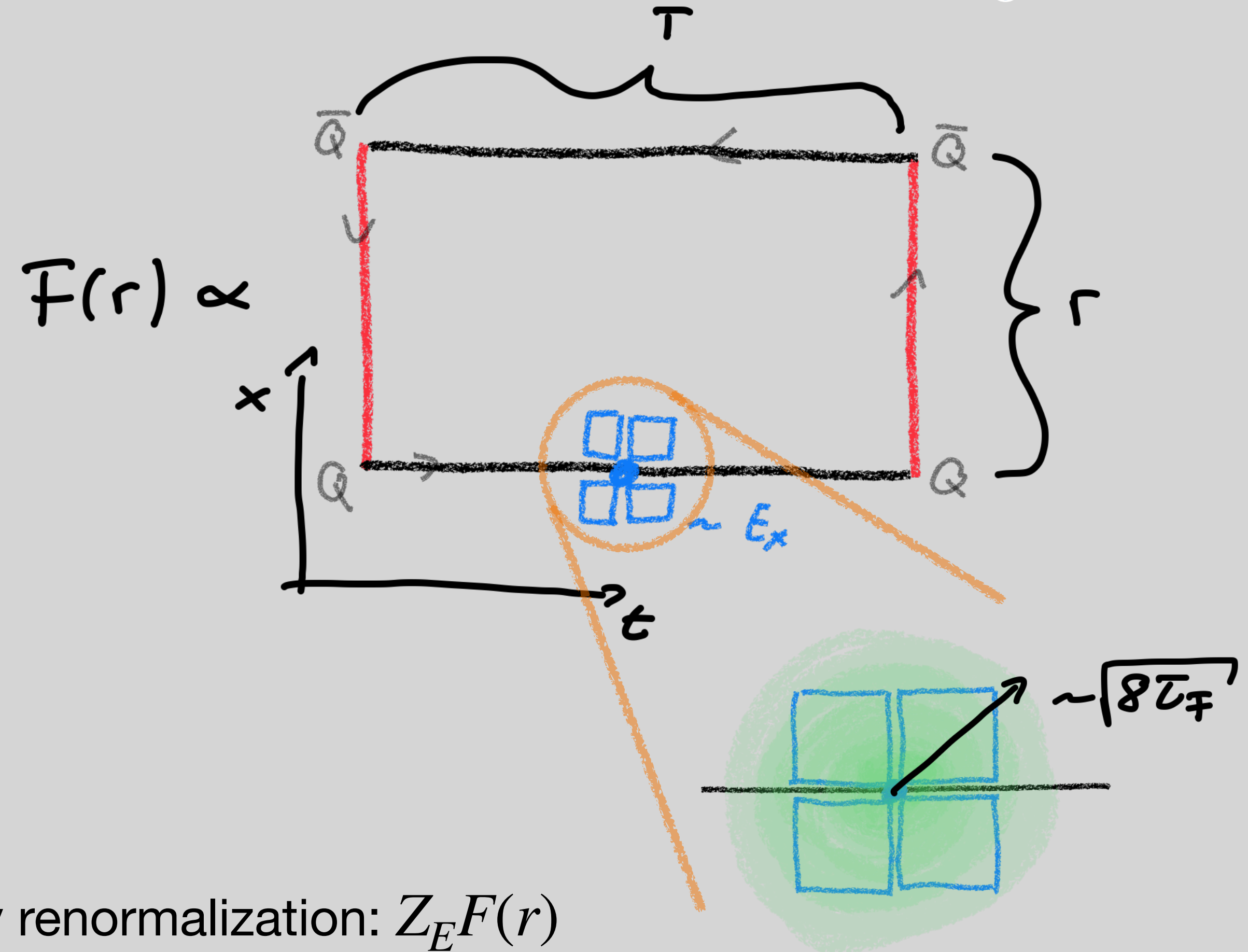
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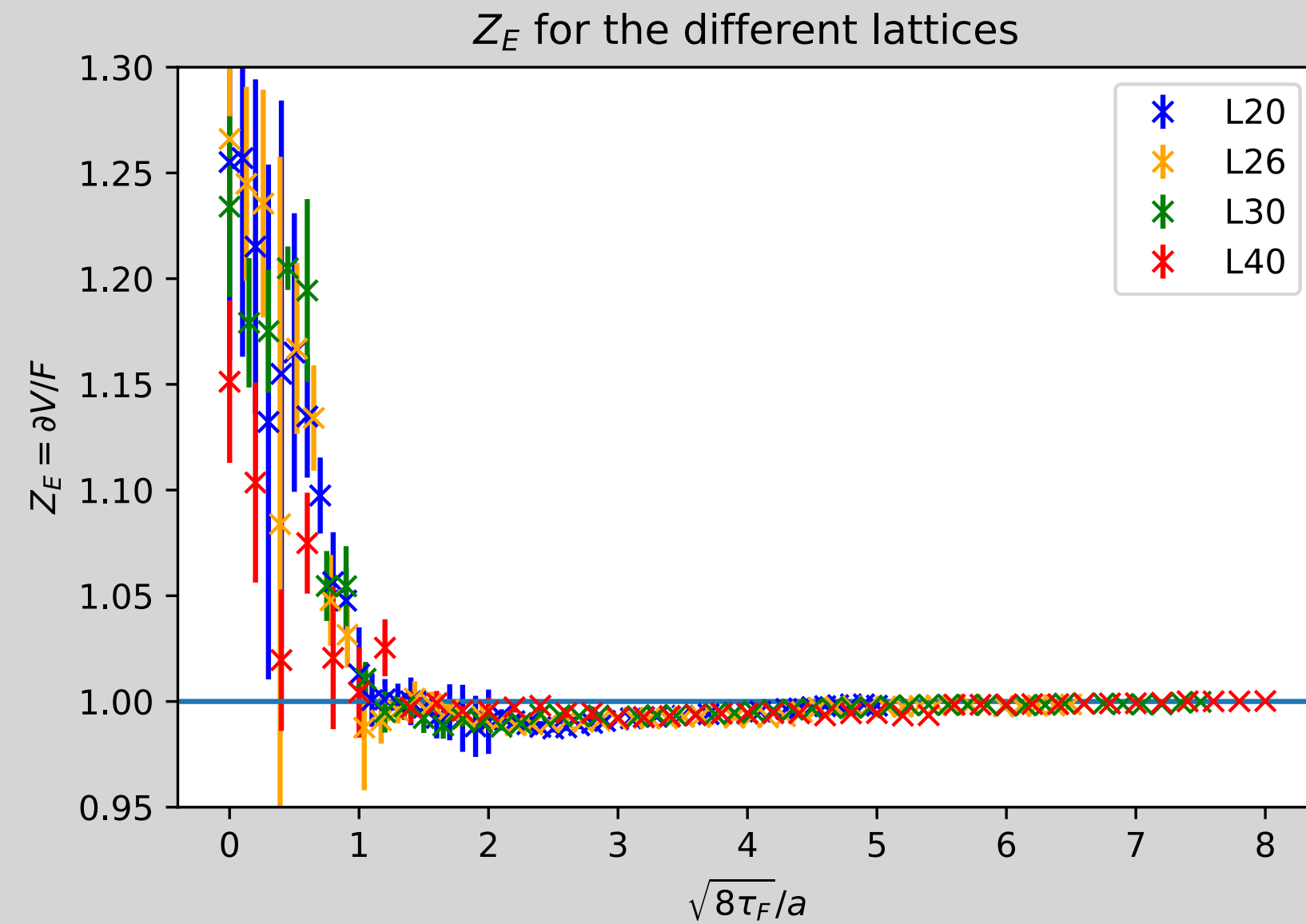


We use gradient flow to improve the signal-to-noise ratio
and to renormalize the observable

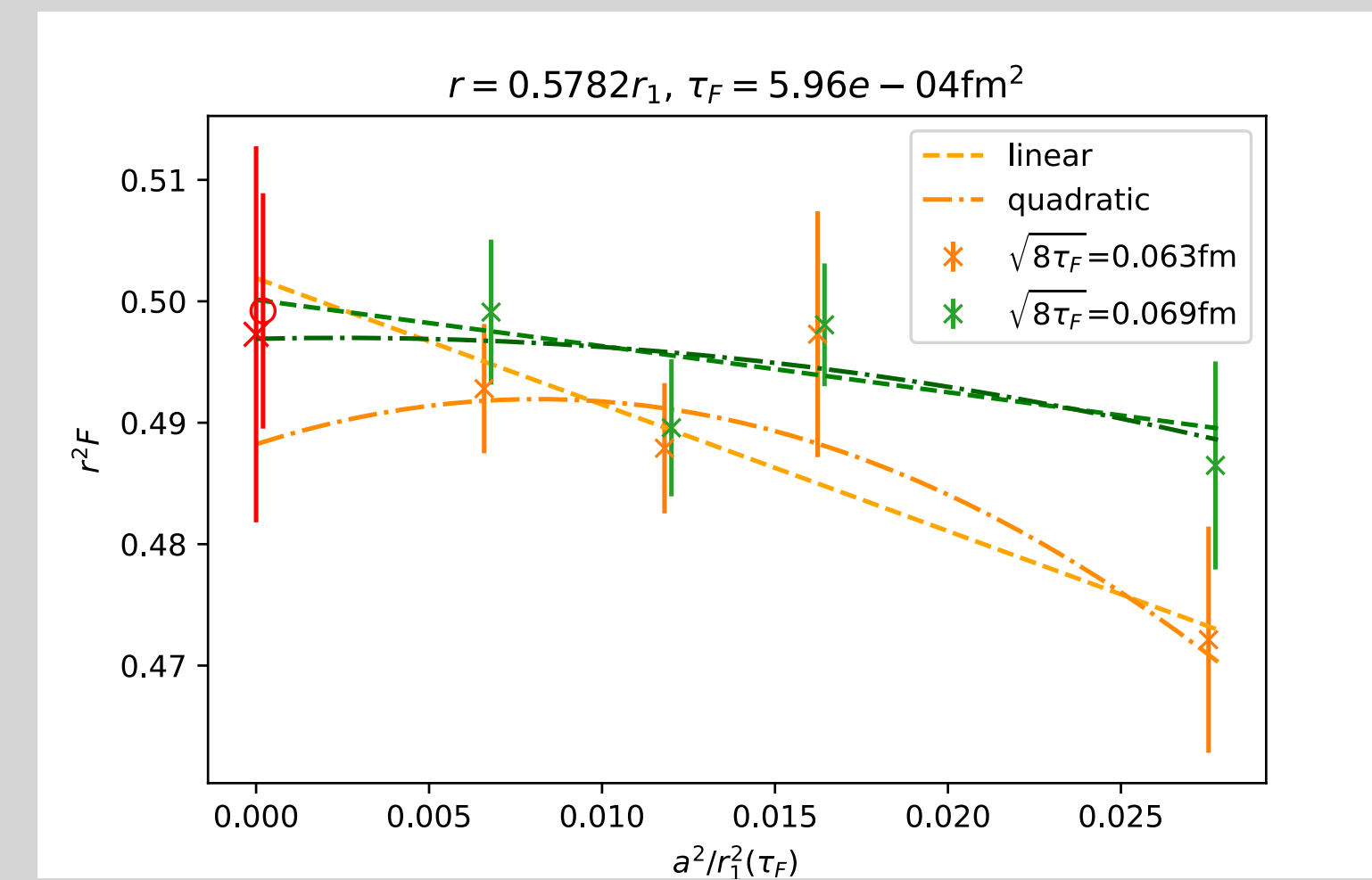
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Renormalization



Continuum limit

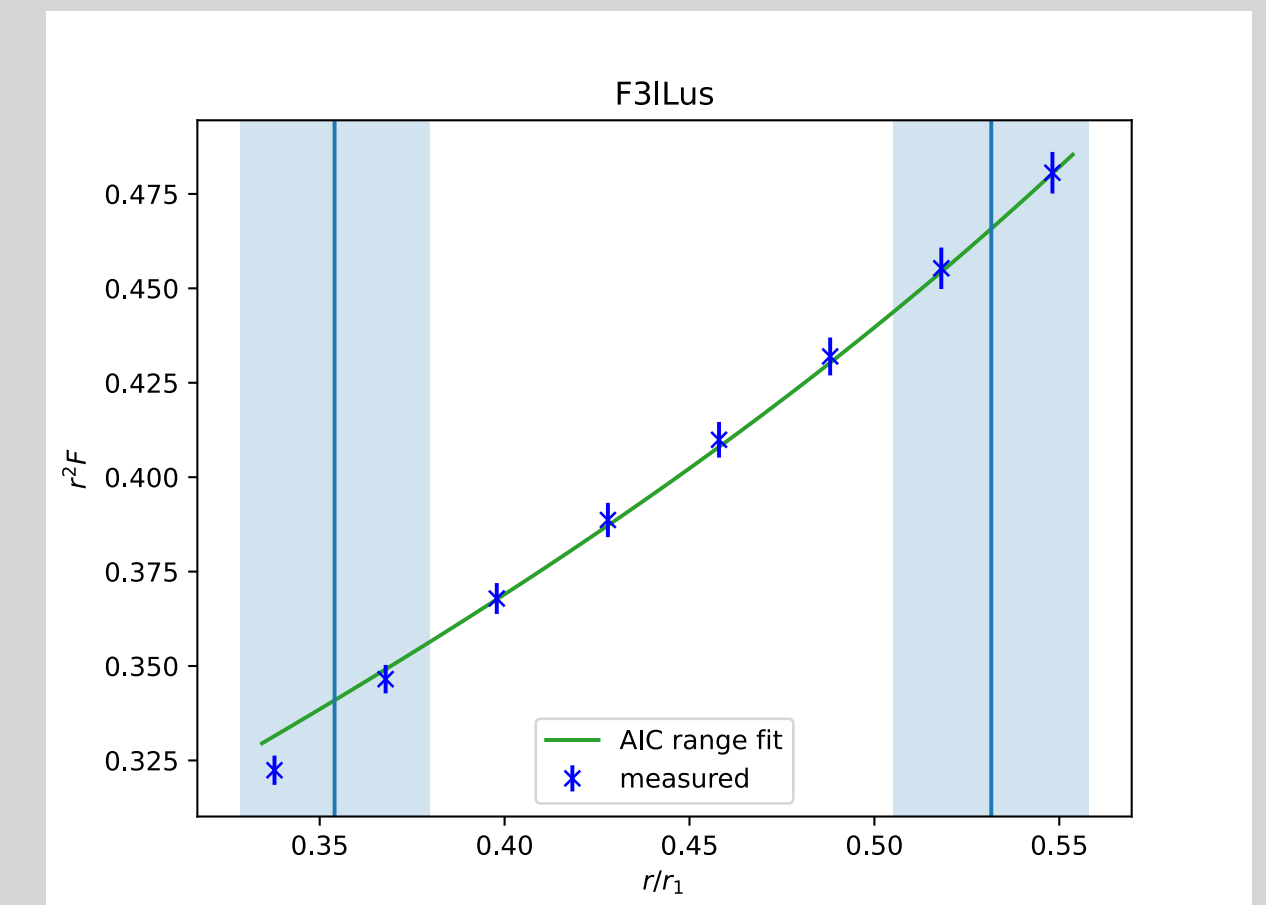
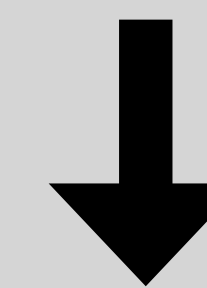
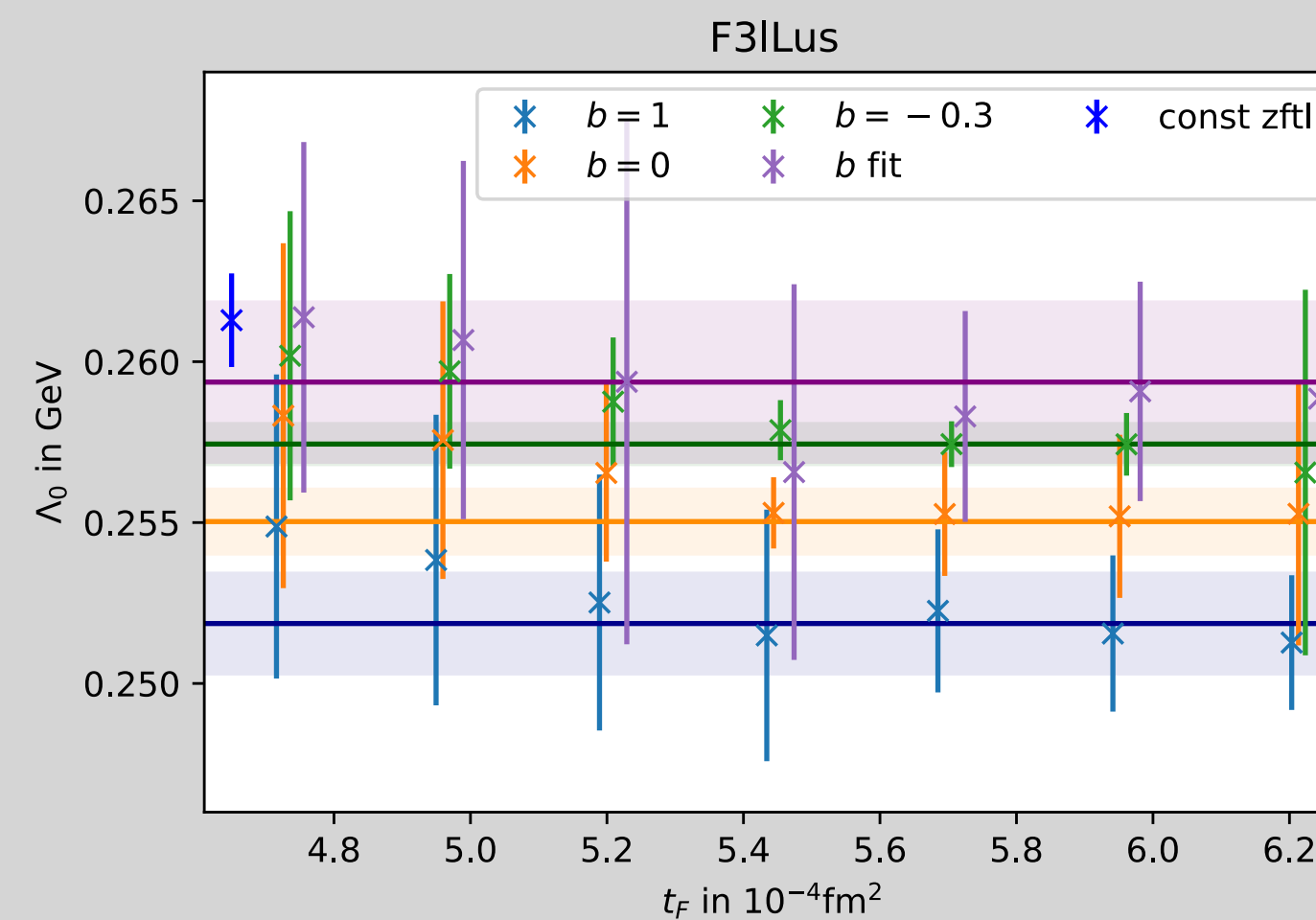
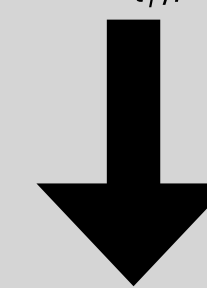
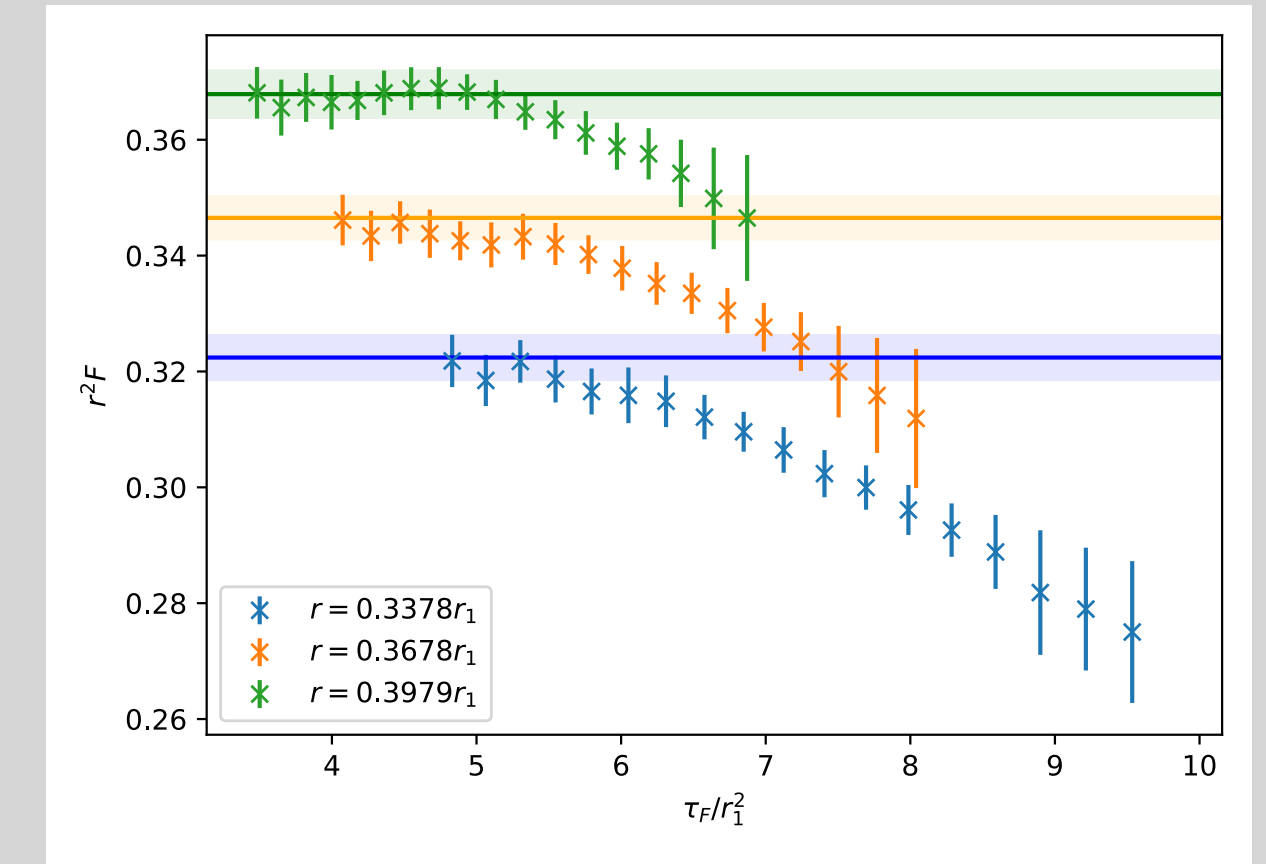
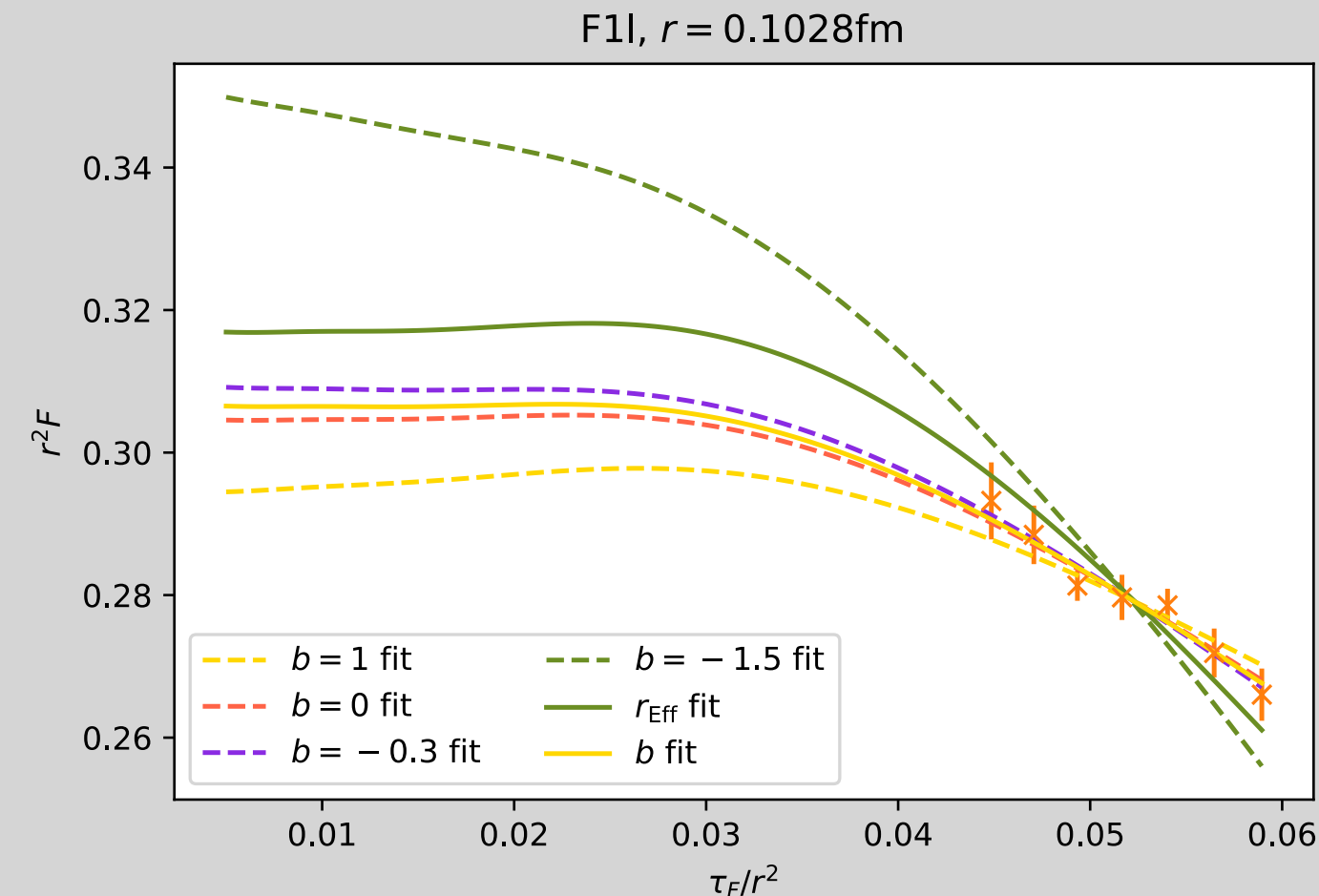


- Gradient flow drives $Z_E \rightarrow 1$ (renormalization)
- Performing proper continuum limit where $Z_E \approx 1$
- Compare the continuum lattice results with perturbative results

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- Comparing with perturbative results:
 - At finite flow time $\rightarrow \Lambda_0(\tau_F)$
 - Perform zero flow time first and then find $\rightarrow \Lambda_0(\tau_F = 0)$
- Static force at finite flow time known only up to 1-loop (combine zero flow time with 1-loop finite flow time knowledge)
- At zero flow time up to N3LL
- At finite flow time:
 - N3LL: $\Lambda_0 = 0.250 \dots 0.262$ GeV
- At zero flow time:
 - N3LL: $\Lambda_0 = 0.261(2)$ GeV



Preliminary