

Charm quark mass using a massive renormalisation scheme

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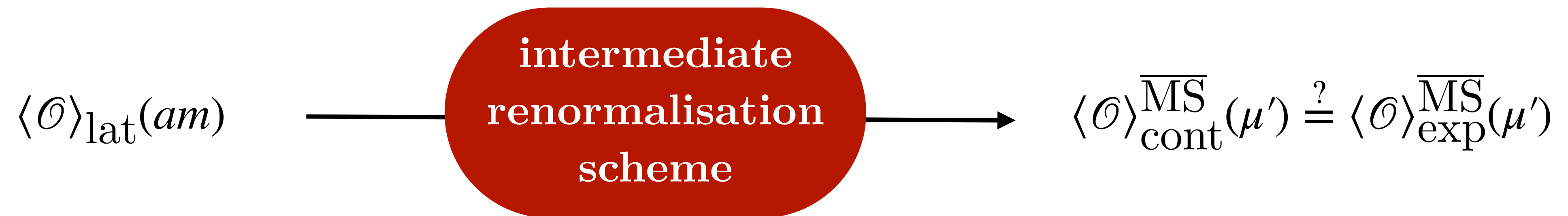
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(U. Southampton)

J T Tsang
(CERN)

Intermediate renormalisation

$$\langle \mathcal{O} \rangle_{\text{lat}}(am) \longrightarrow \langle \mathcal{O} \rangle_{\text{cont}}^{\overline{\text{MS}}}(\mu') \stackrel{?}{=} \langle \mathcal{O} \rangle_{\text{exp}}^{\overline{\text{MS}}}(\mu')$$

Intermediate renormalisation



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$$\langle \mathcal{O} \rangle_{\text{lat}}(am)$$



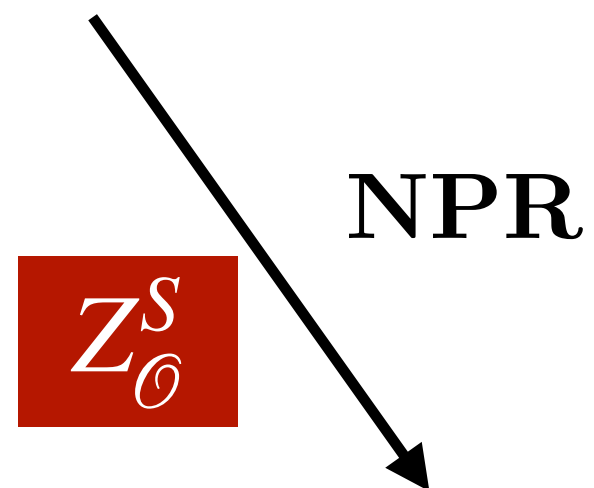
NPR

$$\langle \mathcal{O} \rangle_{\text{lat}}^S(am, a\mu)$$

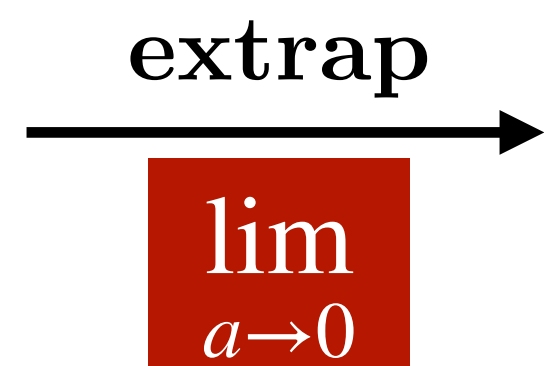
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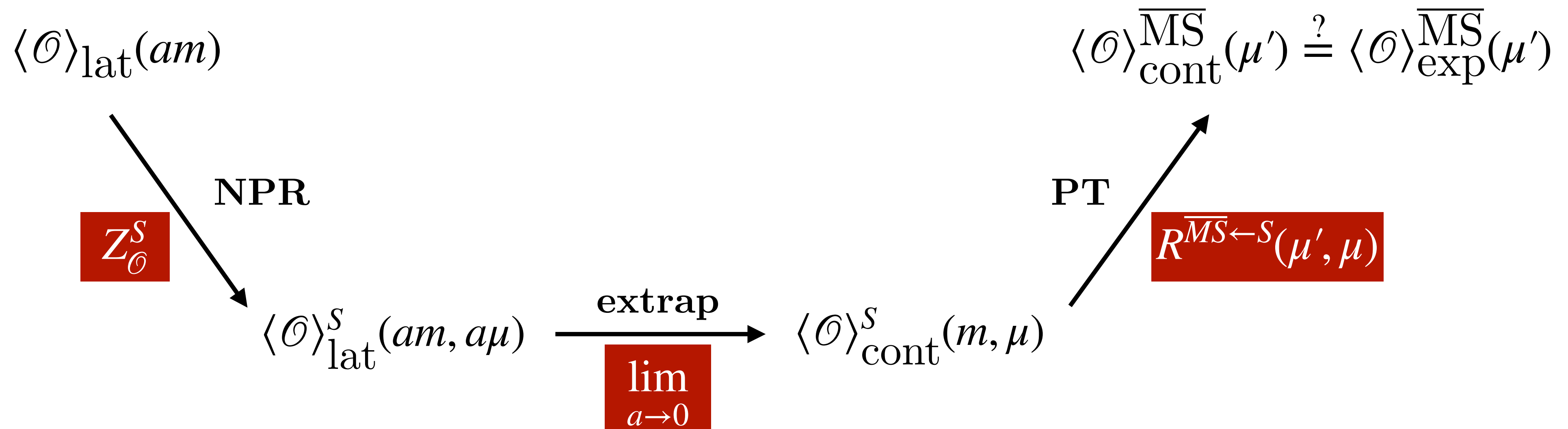
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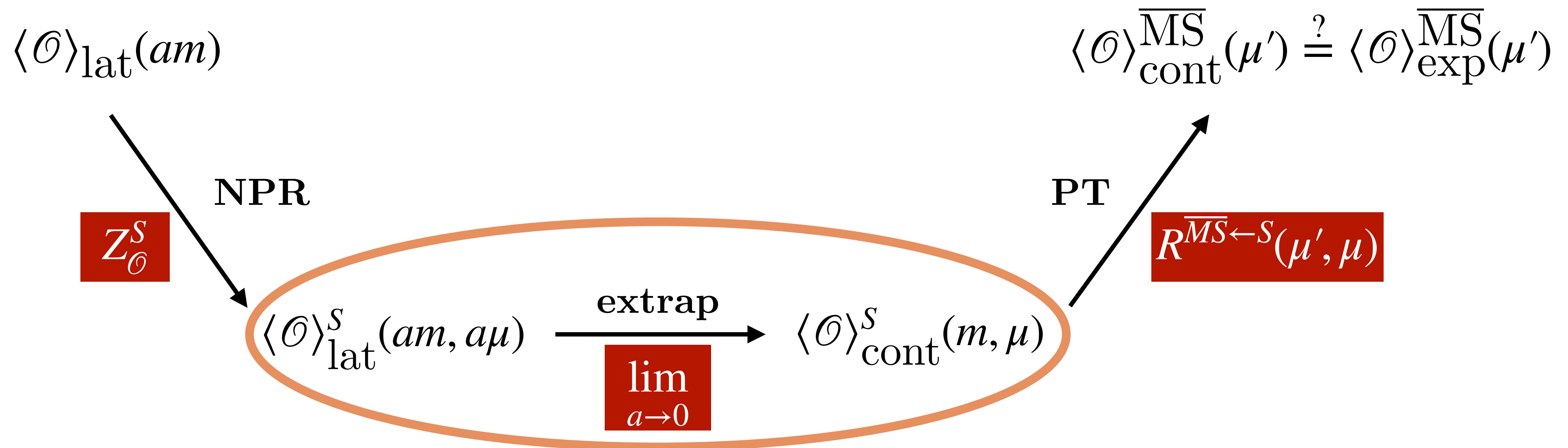
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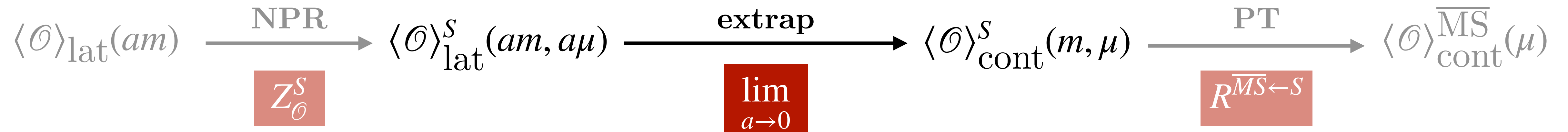
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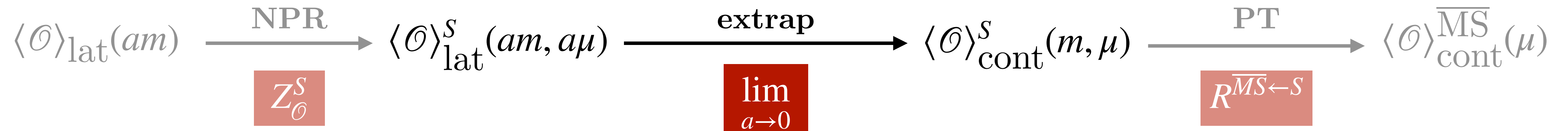


Intermediate renormalisation



$$\langle \mathcal{O} \rangle_{\text{lat}}^S(am, a\mu) = \langle \mathcal{O} \rangle_{\text{cont}}^S(m, \mu) \left[1 + \hat{\delta}(am) + \dots \right]$$

Intermediate renormalisation

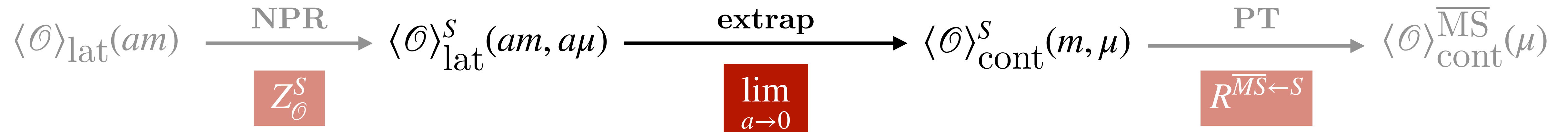


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LATTICE ARTEFACTS
we would like

$$\lim_{a \rightarrow 0} \hat{\delta}(am) = 0$$

Intermediate renormalisation

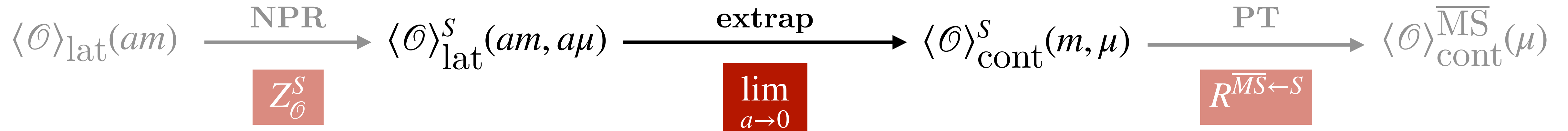


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$$\lim_{a \rightarrow 0} \hat{\delta}(am) \lesssim O(a^2)$$

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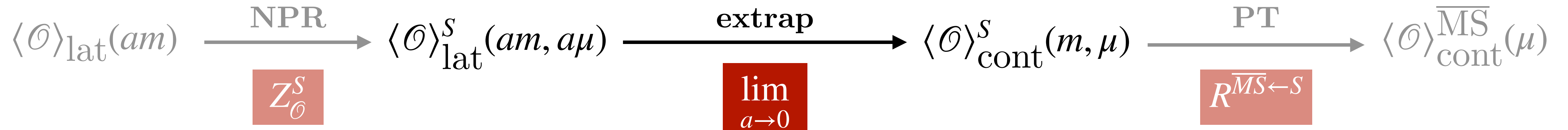


$$\begin{aligned}
 \langle \mathcal{O} \rangle_{\text{lat}}^S(am, a\mu) &= \langle \mathcal{O} \rangle_{\text{cont}}^S(m, \mu) \left[1 + \hat{\delta}(am) + \dots \right] \\
 &= Z_{\mathcal{O}}^S(am, a\mu) \langle \mathcal{O} \rangle_{\text{lat}}(am)
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Choose scheme S such that $Z_{\mathcal{O}}^S$ absorbs lattice artefacts!

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Massless v massive schemes

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Charm quark mass $am_C \not\approx 0$

RBC/UKQCD ($N_f = 2+1$ DWF+I)
ensembles:

$$am_C \approx \begin{cases} 0.56 & \text{Coarse} \\ 0.33 & \text{Medium} \\ 0.27 & \text{Fine} \end{cases}$$

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Renormalised charm quark mass

$$m_{C,\text{cont}}^S(m_R, \mu) = \lim_{a \rightarrow 0} Z_m^S(am, a\mu) \Big|_{m_R} m_{C,\text{lat}}$$

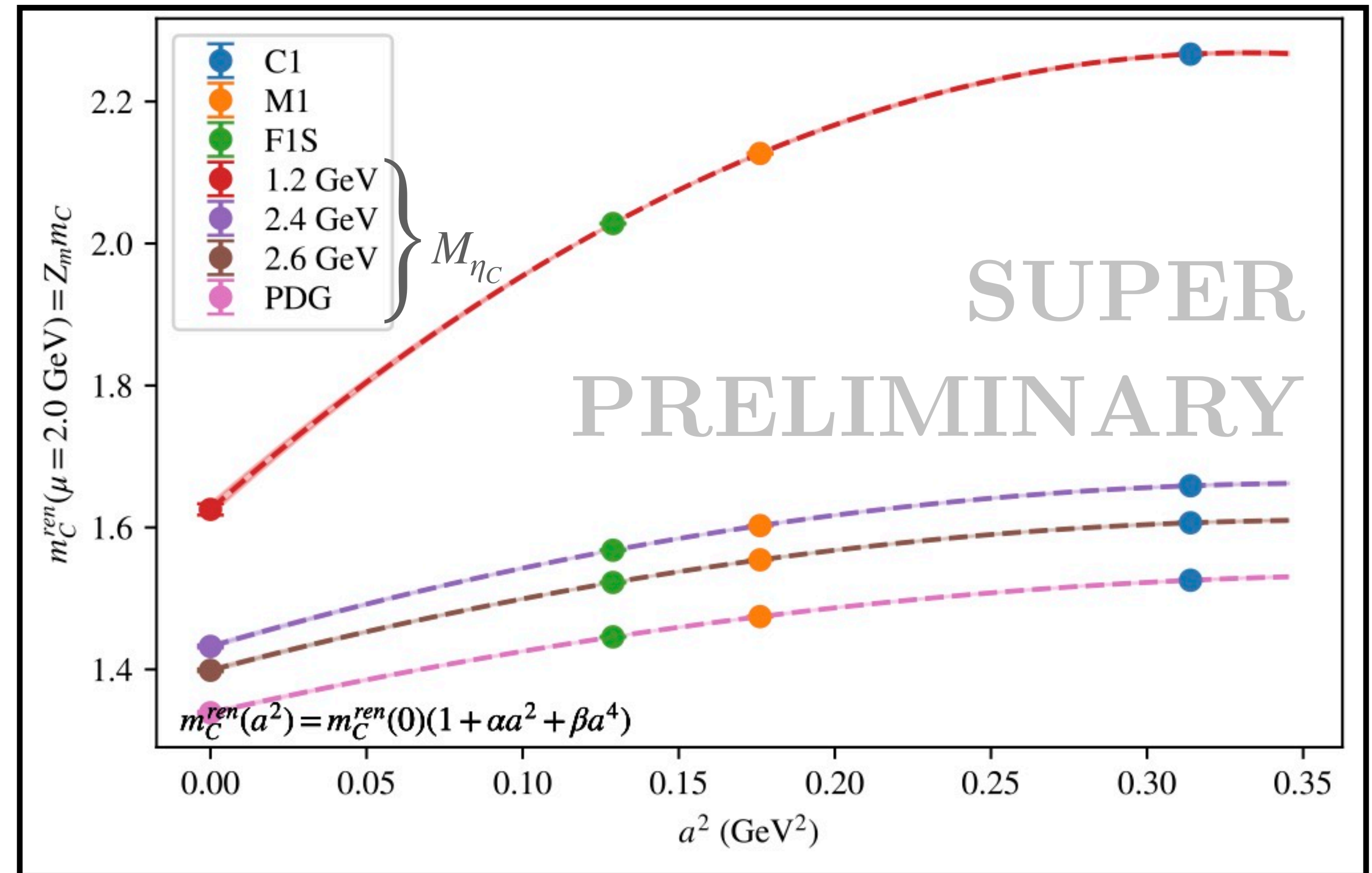
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Things to worry about: $m_C^{\overline{\text{MS}}}$, am_{res} (DWF)
WORK IN PROGRESS!

