Fighting the sign problem in a chiral random matrix model with contour deformations

Dávid Pesznyák

Eötvös Loránd University

in collaboration with Matteo Giordano, Attila Pásztor and Zoltán Tulipánt

[arXiv:hep-lat/2301.12947]

Methods of Effective Field Theory and Lattice Field Theory - Flash Talk Session
July 14, 2023, Bad Honnef Physics School, Germany

Sign problem

Partition function:

$$\mathcal{Z}_w = \int \mathcal{D}\phi \; e^{-S[\phi]} = \int \mathcal{D}\phi \; w[\phi] \; .$$

If $w[u] \notin \mathbb{R}^+$: **complex action problem**.

Reweighting from a theory with positive real weights $r[\phi]$:

$$\langle \mathcal{O} \rangle_w = \frac{\int \mathcal{D}\phi \; \mathcal{O}[\phi] w[\phi]}{\int \mathcal{D}\phi \; w[\phi]} = \frac{\langle \mathcal{O} \frac{w}{r} \rangle_r}{\langle \frac{w}{r} \rangle_r} \; .$$

Complex action problem $\implies$ **sign problem**:

- large fluctuations in w/r $\longrightarrow$ large cancellations $\longrightarrow$ large uncertainties
- severity of the sign problem:

$$\left\langle \frac{w}{r} \right\rangle_r = \frac{\mathcal{Z}_w}{\mathcal{Z}_r} \implies \begin{cases} 1 & \sim \text{perfect!} \\ \approx 0 & \sim \text{not so much\ldots} \end{cases}$$

Fighting the sign problem with contour deformations

Deforming integration manifold $\mathcal{M} \to \mathcal{M}_{\text{def}}(\{p\})$:

$$\mathcal{Z}_w = \int_{\mathcal{M}} \mathcal{D}\phi \, w[\phi] = \int_{\mathcal{M}_{\text{def}}} \mathcal{D}\phi_{\text{def}} \, w[\phi_{\text{def}}] = \int_{\mathcal{M}_{\text{def}}} \mathcal{D}\phi \, \det \mathcal{J}(\phi) w[\phi_{\text{def}}(\phi)] \, .$$

Phase-quenched ($r = |w|$) partition function:

$$\mathcal{Z}^{\text{def}}_{|w|}(\{p\}) = \int_{\mathcal{M}_{\text{def}}} \mathcal{D}\phi \, \Big| \det \mathcal{J}(\phi) w[\phi_{\text{def}}(\phi)] \Big| \, .$$

Severity of sign problem: $\mathcal{Z}_w / \mathcal{Z}^{\text{def}}_{|w|}(\{p\}) = \langle e^{i\theta} \rangle$.

Deformation for larger $\langle e^{i\theta} \rangle \sim$ **milder sign problem**