Exercises

Gamma matrix convention that you need to use to utilize these propagators and correlators:

https://theory.tifr.res.in/~nilmani/BH_school/Gamma_convention.nb

1. Perform Wick contraction for the following operator corresponding to the proton $({}^1/_2)^+$:

$$N_\alpha(x):\ \epsilon_{abc}\left(u^T(x)\right)^a_\beta (C\gamma_5)_{\beta\gamma}\left(d(x)\right)^b_\gamma\left(u(x)\right)^c_\alpha \quad C:\ \gamma_2\gamma_4$$

Take the spin-projection operator to be $P = (1+\gamma_4)$
Code the two-point function with the provided quark propagator $(m_u = m_d)$

Quark prop link: https://theory.tifr.res.in/~nilmani/BH_school/quark_prop1 (lattice size $4^3 \times 8$)
https://theory.tifr.res.in/~nilmani/BH_school/quark_prop24q64 (lattice size $24^3 \times 64$)

A c-program to read the quark propagators: https://theory.tifr.res.in/~nilmani/BH_school/prop_read.c

2. Two correlators for a $64^3 \times 192$ lattice are provided. They have 142 configurations.
   Data (ascii) is in the following format

```
open loop over number of configurations (=142)
open loop over number of time slices (=192)
          junk, junk, time slice,  correlator value, 0
 close loop over number of time slices
 close loop over number of configurations
```

Correlator link: https://theory.tifr.res.in/~nilmani/BH_school23/correlator64_1
https://theory.tifr.res.in/~nilmani/BH_school23/correlator64_2

Fit these correlators to get their ground state masses (m_1 and m_2). Calculate their splitting $\Delta m = m_1 - m_2$ from direct fitting and ratio fitting. If $a = 0.044065$ fm, calculate Δm in the unit of MeV. From the splitting information can you guess which meson particles may correspond to these two correlators?

Exercises

3. Construct an operator for a state with the quantum number $\boldsymbol{J^{PC} = 2^{++}}$

4. Consider a 1D potential of the form: $(x^4 - 3)e^{-x^2/2}$

   Assume this potential inside a periodic box of size L = 10 in between $-5 \leftrightarrow +5$. Solve the Schrödinger equation numerically and calculate the first 20 energy levels. Next change the box size from 10 ($-5 \leftrightarrow +5$) to 50 ($-25 \leftrightarrow +25$) (in an increment of 1) and calculate again the first 20 energy levels for each box size. Then plot these energy levels as a function of box size L. Observe the pattern of the third energy level in positive parity. Repeat the solution for negative parity energy levels.

5. Consider the following interpolating operators

$$M_i = \bar{Q}\gamma_5 q_i, \qquad i = 1,2$$

$$M_i = \bar{Q}\gamma_k q_i, \qquad i = 1,2; k = 1,2,3$$

$$M = M_1 M_2^* - M_2 M_1^*$$

Quark prop link:
https://theory.tifr.res.in/~nilmani/BH_school/quark_prop1
https://theory.tifr.res.in/~nilmani/BH_school/quark_prop2
https://theory.tifr.res.in/~nilmani/BH_school/quark_prop3

(lattice size $4^3 \times 8$)

Perform the Wick contractions to obtain $\langle M_i M_i^\dagger \rangle, \left\langle M_i^* M_i^{*\dagger} \right\rangle$, and $\langle M M^\dagger \rangle$

Code these two-point functions with the provided quark propagators for $S(q_1), S(q_2)$, and $S(Q)$

Plot $\langle M M^\dagger \rangle$ and $\langle M_1 M_1^\dagger \rangle * \left\langle M_2^* M_2^{*\dagger} \right\rangle$ together as a function of time slices