Nuclear Physics from EFT. Exercises

Problem 1:

The nucleon-nucleon chiral Lagrangian contains two independent operators

$$\mathcal{L}_{NN} = -\frac{C_S}{2}\bar{N}N\,\bar{N}N - \frac{C_T}{2}\bar{N}\vec{\sigma}N\,\cdot\bar{N}\vec{\sigma}N$$

Using the Fierz relations for Pauli matrices,

$$\begin{aligned}\sigma^i_{AB}\,\sigma^i_{CD} &= \frac{3}{2}\delta_{AD}\delta_{CB} - \frac{1}{2}\sigma^i_{AD}\,\sigma^i_{CB}\\ \delta_{AB}\,\delta_{CD} &= \frac{1}{2}\delta_{AD}\delta_{CB} + \frac{1}{2}\sigma^i_{AD}\,\sigma^i_{CB}\end{aligned}$$

in spin and isospin space, show that the operators

$$C_\tau\bar{N}\boldsymbol{\tau}N\,\cdot\bar{N}\boldsymbol{\tau}N \qquad \text{and} \qquad C_{\sigma\tau}\bar{N}\boldsymbol{\tau}\vec{\sigma}N\cdot\,\bar{N}\boldsymbol{\tau}\vec{\sigma}N$$

can be expressed in terms of C_S and C_T.

Problem 2:

Consider nucleons and pions interacting by the leading order CP-even chiral Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + \bar{N}\left\{iv\cdot\partial - \frac{1}{4F_\pi^2}(\boldsymbol{\pi}\times v\cdot\partial\boldsymbol{\pi})\cdot\boldsymbol{\tau} - \frac{g_A}{F_\pi}S\cdot\partial(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\right\}N + \dots$$

and by a small CP-odd pion-nucleon interaction

$$\mathcal{L}_{CP} = -\frac{\bar{g}_0}{F_\pi}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N$$

(a) Derive the Feynman rules for the πNN and $\pi\pi NN$ vertices

(b) Derive the CP-even and CP-odd one-pion-exchange potentials, in the convention in which the Born contribution to the scattering amplitude is given by $i\mathcal{A} = -i\langle\vec{p}\,'|V|\vec{p}\rangle$, with $\vec{p}$ ($\vec{p}\,'$) the relative momentum of the incoming (outgoing) nucleons.

(c) Draw all the two-pion-exchange diagrams linear in $\bar{g}_0$, and write down their expressions using HBχPT Feynman rules.

(d) Identify the diagrams that give rise to "pinch singularities" and show that for these diagrams the matching between HBχPT and chiral EFT can be accomplished by replacing

$$\frac{1}{-v\cdot k + i\varepsilon} = -\frac{1}{v\cdot k + i\varepsilon} - 2\pi i\delta(v\cdot k)$$

and by neglecting the delta function.

(e) Calculate the CP-odd two-pion-exchange potential. You might find convenient to use "λ parameters", in addition to standard Feynman parameters:

$$\frac{1}{(v\cdot k+i\varepsilon)^a}\frac{1}{[k^2+\Delta^2+i\varepsilon]^b}=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^{+\infty}d\lambda\frac{2^a\lambda^{a-1}}{[k^2+\Delta^2+2\lambda v\cdot k]^{a+b}}$$

(f) What is the form of the short-range CP-odd operators needed to absorb the UV divergences in the two-pion-exchange potential?

Problem 3:

Consider a universe in which a neutron and a proton in the ${}^1S_0$ state form a bound state, with $B=-3$ MeV. The pion exchange potential in this channel is

$$V_\pi(r)=-\frac{g_A^2m_\pi^2}{16\pi F_\pi^2}\frac{e^{-m_\pi r}}{r}.$$

Regulate the delta-function potential with a coordinate-space Gaussian regulator

$$V_\delta(r,R_S)=C_{{}^1S_0}(R_S)\frac{1}{\pi^{3/2}R_S^3}\exp\left(-\frac{r^2}{R_S^2}\right).$$

(a) Choose a grid of values of the cut-off R_S between 0.5 and 0.05 fm (or smaller). Consider the Schroedinger equation for the radial function of the bound state

$$\frac{d^2}{dr^2}y(r)=\left(\gamma^2+V_\pi(r)+V_\delta(r,R_S)\right)y(r),\qquad \text{with}\qquad \psi(\vec{r})=\frac{y(r)}{r}Y_{00}(\theta,\varphi),\tag{1}$$

and with $\gamma^2=-m_NB$. For each R_S, determine the value of the coefficient $C_{{}^1S_0}(R_S)$ for which Eq. (1) has solution, and find the bound state wavefunction. (*Hint:* you can use the two-point boundary condition ODE solver `scipy.integrate.solve_bvp`.)

(b) Plot the wavefunction and $C_{{}^1S_0}$. How does $C_{{}^1S_0}$ depend on R_S?

(c) In this universe, the photon is massive, $m_\gamma=200$ MeV, and couples to the neutron with charge $Q_n=1$, while the proton has $Q_p=0$. The fine structure constant is $\alpha=1/137$. Find the leading-order one-photon-exchange potential between two neutrons.

(d) For each value of R_S, calculate the shift in the dineutron binding energy with respect to the np binding energy, at leading order in α. Plot $\gamma_{nn}^2-\gamma_{np}^2$ as a function of R_S in a log-linear plot and determine an extrapolation formula. Knowing γ_{np}^2, m_γ, and α, can you predict

$$\lim_{R_S\to 0}\left(\gamma_{nn}^2(R_S)-\gamma_{np}^2\right)$$

with no additional experimental input?