Indirect Methods in Nuclear Astrophysics

Introduction to Theoretical Nuclear Physics and Nuclear Astrophysics

Lectures 5 & 6

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Outline

- **Motivation**
  nuclear reactions of astrophysical interest, types of reactions,
  reactions rates, cross sections, charged-particle reactions, electron screening
- **Indirect Methods**
  overview, general characteristics
- **Reaction Theory**
  degrees of freedom, cross sections, potential scattering, T matrix,
  approximations, spectroscopic factors
- **Coulomb Dissociation Method**
  idea, semiclassical/quantal theory, characteristic parameters,
  example, higher-order effects
- **Wave Functions**
  asymptotics, cross sections, penetrability factor, radial integrals
- **ANC Method**
  idea, theory, example, effects of continuum interaction
- **Trojan-Horse Method**
  idea, theory, application, examples
- **Conclusions**

Motivation I - Nuclear Reactions of Astophysical Interest

nuclear astrophysics

- nuclear reaction rates are basic input in many astrophysical models
  (primordial nucleosynthesis, stellar evolution, novae, supernovae, . . .)
  for various processes (pp chains, CNO cycles, s-, r-, p-, rp-process, . . .)
- ideally: direct measurement of reaction cross sections at relevant energies
  but in most cases practically impossible (small cross sections, often unstable nuclei)
- alternative approaches ?
  depend on type of reaction

Motivation II - Types of Reactions

- radiative capture/
  photo dissociation reactions:

  (n,γ), (p,γ), (α,γ), ... /
  (γ,n), (γ,p), (γ,α), ...

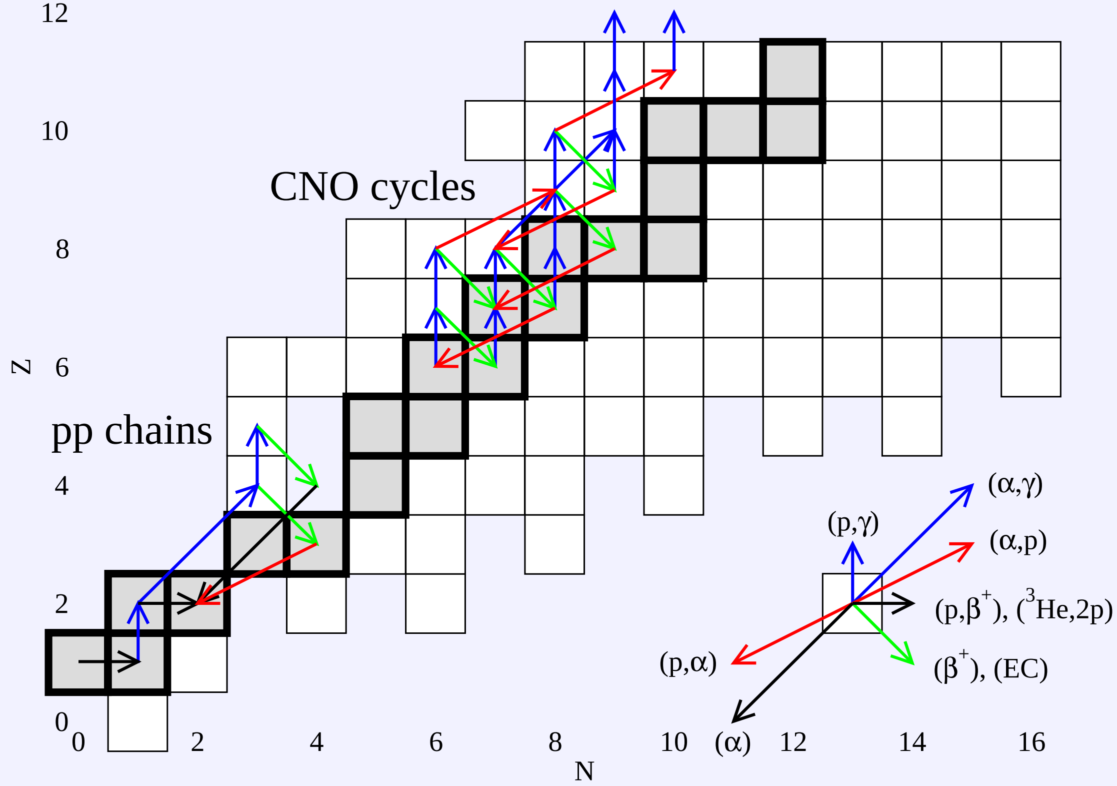
- nuclear rearrangement reactions:

  (p,α), (α,p), $(^3\text{He},2p)$, ...

- weak interaction reactions:

  β^+, β^-, electron capture (EC)

- ...



here:

- only charged-particle reactions
- only with electromagnetic or strong interaction

Motivation III- Reaction Rates and Cross Sections

astrophysical environment

nuclei in hot plasma

⇒ temperature-dependent distribution of relative velocities v for reaction $b + x \to \dots$

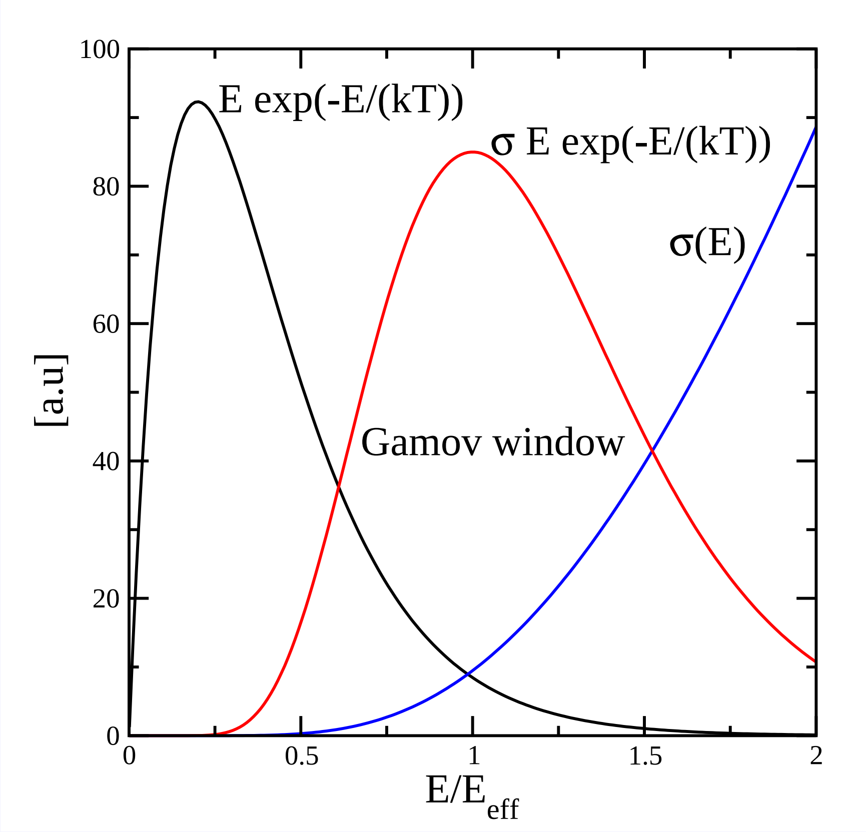
⇒ relevant quantity:

Maxwellian-averaged reaction rate

$$r_{bc} = \frac{\varrho_b \varrho_x}{1+\delta_{bx}} \langle \sigma v \rangle$$

with densities ϱ_b, ϱ_x and

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu_{bx}}} \int_0^\infty \sigma(E)\; E\; e^{-\frac{E}{kT}} \frac{dE}{(kT)^{3/2}}$$



⇒ cross sections σ needed in Gamov window of width ΔE around effective energy E_{eff}

Motivation IV- Gamov Window

parameters

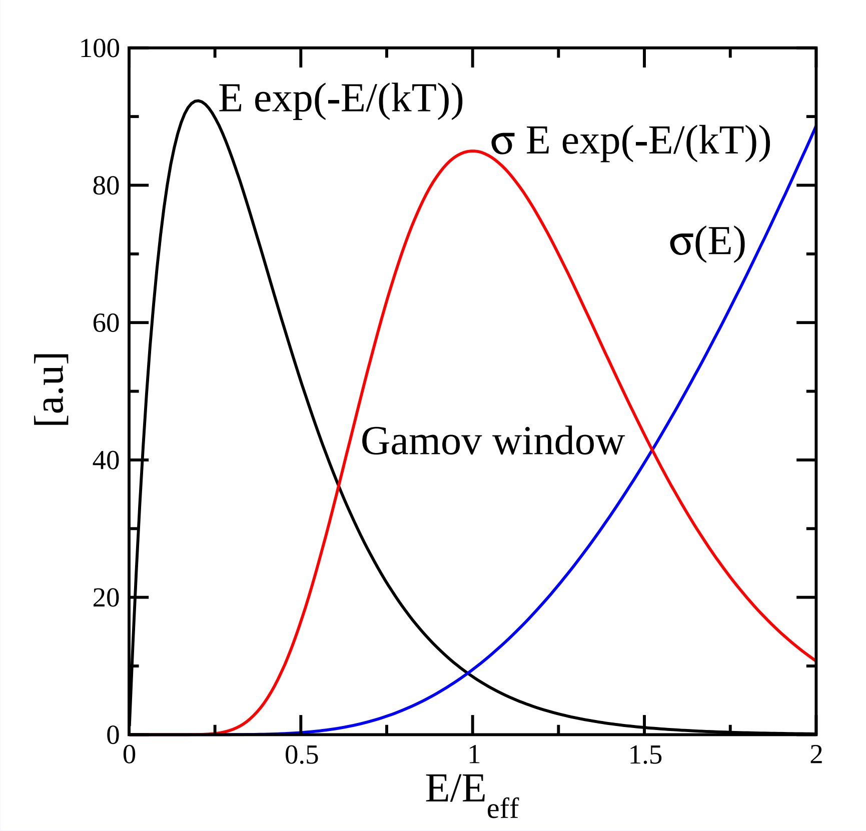
- effective energy

$$E_{\text{eff}} = 0.1220\, \mu_{bx}^{1/3} (Z_b Z_x T_9)^{2/3} \text{ MeV}$$

- width

$$\Delta E = 0.2368\, \mu_{bx}^{1/6} (Z_b Z_x)^{1/3} T_9^{5/6} \text{ MeV}$$

with temperature T_9 in 10^9 K
and reduced mass $\mu_{bx} = \frac{m_b m_x}{m_b + m_x}$ in amu



reaction	E_{eff} [keV]	$\sigma(E_{\text{eff}})$ [pb]
^{3}He(^{3}He,2p)^{4}He	22.0	1.5
^{7}Be(p,γ)^{8}B	18.4	1.5×10^{-3}
^{3}He(α,γ)^{7}Be	23.0	3.0×10^{-5}
^{14}N(p,γ)^{15}O	27.2	2.2×10^{-7}

for $T = 15.5 \times 10^6$ K (center of the sun)

Motivation V - Charged-Particle Reactions

- **Coulomb barrier** in reaction $b + x \rightarrow \ldots$ with charged nuclei b, x

  $\Rightarrow$ extremely small cross sections $\sigma(E)$ with strong energy dependence

  $\Rightarrow$ astrophysical relevant energies (Gamov window) usually not accessible

  $\Rightarrow$ measurement at higher energies and extrapolation to low energies E

  with help of astrophysical S factor $S(E) = \sigma(E) E \exp(2\pi\eta)$

  Sommerfeld parameter $\eta = Z_b Z_x e^2/(\hbar v)$

  $\Rightarrow$ danger of extrapolation error, missed resonances, bound state tails

- direct measurement very difficult, often unstable nuclei involved

- cross sections of light particle reactions are dominated by non-resonant and only few resonant contributions at small energies

- electron screening in laboratory experiments and in stellar plasma

Motivation VI - Electron Screening

direct experiments:

- reduction of Coulomb barrier by electron cloud of target nucleus
- enhanced cross section at low energies

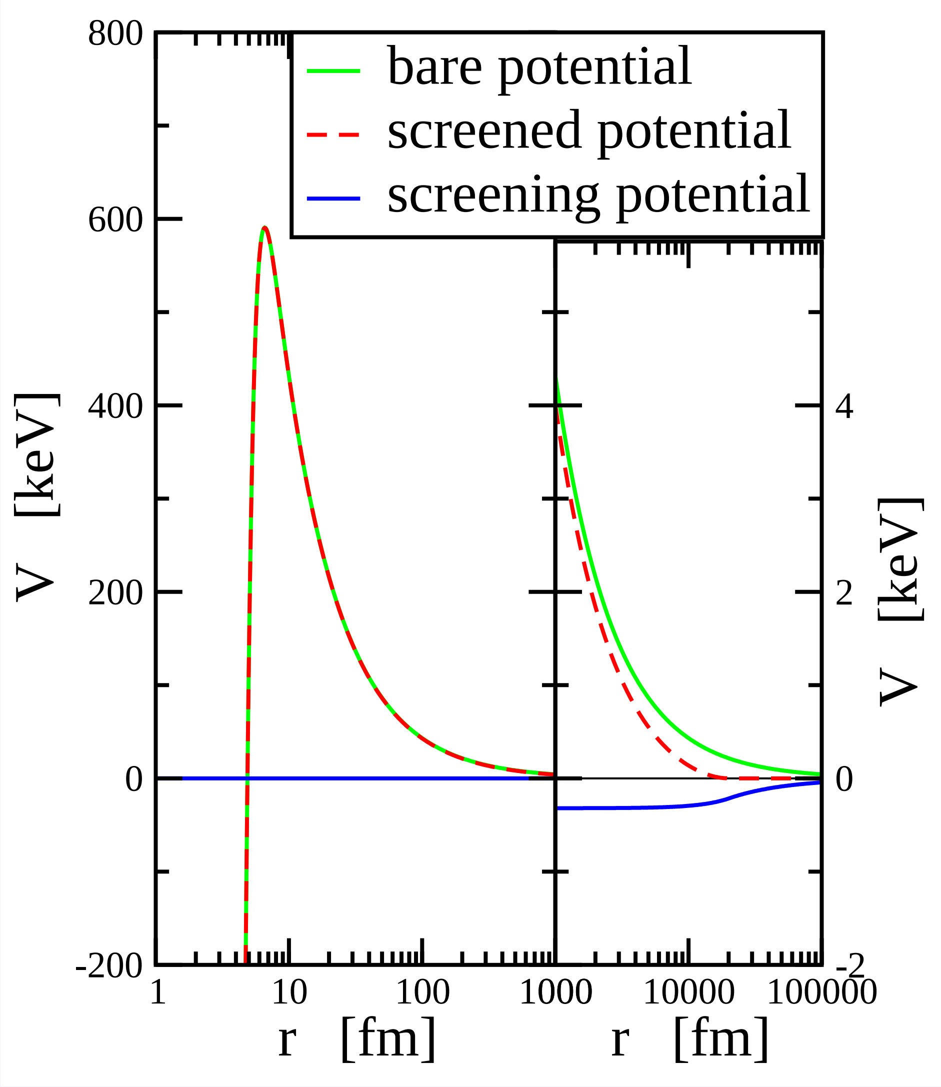
  $\sigma_{\text{exp}}(E) = \sigma_{\text{bare}}(E) f(E)$ with

  $f(E) = \exp(\pi \eta U_e / E)$ and

  electron screening potential energy U_e
- discrepancy between experimental observation and theoretical models, explanation?

stellar conditions:

- electron screening in plasma



Indirect Methods - Overview I

Coulomb dissociation
G. Baur et al.,
NPA 458 (1986) 188

- study inverse of radiative capture reaction
  $b(x,\gamma)a \;\Leftrightarrow\; a(\gamma,x)b$
- use Coulomb field of target nucleus A as source of photons
  $a(\gamma,x)b \;\Leftrightarrow\; A(a,bx)A$

$\Downarrow$

absolute S factors
as a function of energy

ANC method
H. M. Xu et al.,
PRL 73 (1994) 2027

- extract asymptotic normalization coefficient of ground state wave function of nucleus a from transfer reactions
- calculate matrix elements for radiative capture reaction $b(x,\gamma)a$

$\Downarrow$

S factor at zero energy

Trojan-Horse method
G. Baur,
PLB 178 (1986) 35

- study three-body reaction
  $A+a \longrightarrow C+c+b$
  with Trojan horse
  $a = b + x$
  and spectator b
- extract cross section of two-body reaction
  $A+x \longrightarrow C+c$

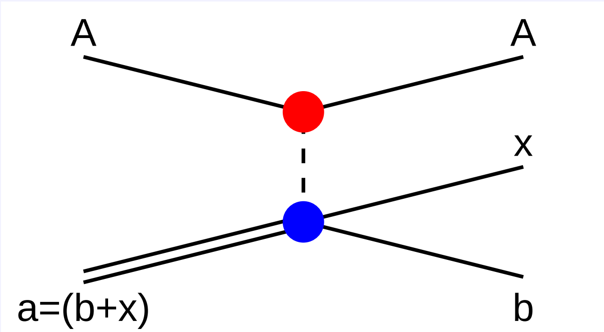
$\Downarrow$

energy dependence
of S factor

theoretical description? relation of methods?

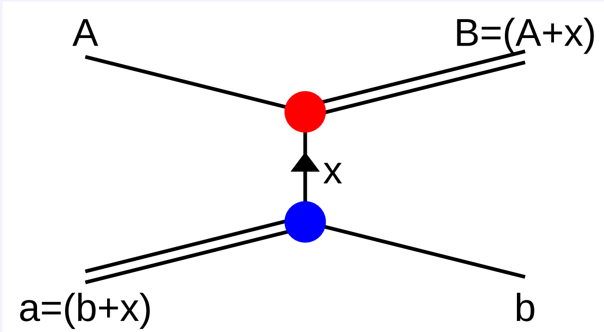
Indirect Methods - Overview II

Coulomb dissociation



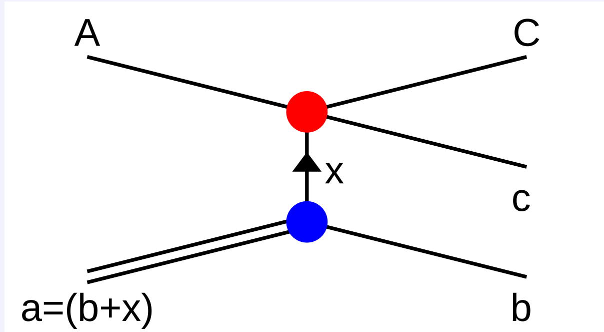
photon exchange

ANC method



transfer of particle to bound state

Trojan-Horse method



transfer of particle to continuum state

- similar reaction mechanisms: transfer of virtual particle
- final state with three particles (bound/continuum states)
- theoretical descripton with direct reaction theory

Indirect Methods - Overview III

general characteristics:

- two-body reaction at low-energy is replaced by three-body reaction at "high-energy" with large cross section
  - ◦ Coulomb dissociation $b(x,\gamma)a \Rightarrow A(a,bx)A$
  - ◦ ANC method $b(x,\gamma)a \Rightarrow A(a,B)b$ $\quad a = (b+x) \quad B = (A+x)$
  - ◦ Trojan-horse method $A(x,c)C \Rightarrow A(a,Cc)b$
- transfer of virtual particle (photon γ or nucleus x)
- relation of cross sections is found with the help of nuclear direct reaction theory
- theoretical approximations essential
- study of peripheral reactions
  - – asymptotics of wave functions relevant
  - – selection of suitable kinematical conditions important

Reaction Theory - Degrees of Freedom

general reaction: $p + t \rightarrow e_1 + e_2 + \ldots + e_n$

- initial state: given momenta of projectile p and target t

  lab system: $\vec{p}_p = p_p \vec{e}_z \qquad \vec{p}_t = 0$

  cm system: $\vec{p}_p = p_p \vec{e}_z \qquad \vec{p}_p + \vec{p}_t = 0$

- final state: determined by momenta of n ejectiles $\vec{p}_j$ $\quad j = 1, \ldots, n$

  $\Rightarrow 3n$ free quantities

- conditions: energy and momentum conservation $\Rightarrow 4$ equations

$\Rightarrow$ $N = 3n - 4$ free variables in final state

particles in final state n	free variable in final state N	physical variables
2	2	two angles $(\vartheta,\ \varphi)$
3	5	one energy and four angles
4	8	two energies and six angles

Reaction Theory - Cross Sections I

general form for reaction $p + t \to e_1 + e_2 + \ldots + e_n$

$$d\sigma = \frac{2\pi}{\hbar}\frac{1}{v_{pt}}\frac{1}{N_i}\sum_{i=1}^{N_i}\sum_{f=1}^{N_f}\int |T_{fi}|^2\,\delta(E_f - E_i - Q)(2\pi\hbar)^3\delta(\vec{P}_f - \vec{P}_i)\prod_{j=1}^{n}\frac{d^3p_j}{(2\pi\hbar)^3}$$

- normalization to flux in initial state with relative velocity $v_{pt} = \frac{p_{pt}}{\mu_{pt}}$
- summation over final states and averaging over initial states (e.g. spin)
- energy conservation with kinetic energies $E_i = \frac{p_p^2}{2m_p} + \frac{p_t^2}{2m_t}$, $E_f = \sum_j \frac{p_j^2}{2m_j}$ and Q value of reaction
- momentum conservation with $\vec{P}_i = \vec{p}_p + \vec{p}_t$, $P_f = \sum_j \vec{p}_j$
- phase space factor
  - transformations between coordinate systems (cm/lab, Jacobi coordinates)
  - integration over unobserved quantities: various differential/total cross sections
- all essential information on reaction mechanism contained in T-matrix element T_{fi}

Reaction Theory - Cross Sections II

example: two-body reaction $p + t \to e_1 + e_2$

- relative and total momenta in initial and final state

$$\vec{p}_{pt} = \mu_{pt}\left(\frac{\vec{p}_p}{m_p} - \frac{\vec{p}_t}{m_t}\right) \qquad \vec{P}_{pt} = \vec{p}_p + \vec{p}_t \qquad \vec{p}_{12} = \mu_{12}\left(\frac{\vec{p}_1}{m_1} - \frac{\vec{p}_2}{m_2}\right) \qquad \vec{P}_{12} = \vec{p}_1 + \vec{p}_2$$

integration over total momentum trivial with $d^3p_1 d^3p_2 = d^3p_{12} d^3P_{12}$

$$\Rightarrow \quad d\sigma = \frac{2\pi}{\hbar}\frac{\mu_{pt}}{p_{pt}}\frac{1}{N_{pt}}\sum_{i=(pt)}\sum_{f=(12)}\int |T_{fi}|^2\,\delta(E_f - E_i - Q)\frac{d^3p_{12}}{(2\pi\hbar)^3}$$

- kinetic energies $E_i = \frac{p_{pt}^2}{2\mu_{pt}} + \frac{P_{pt}^2}{2M_{pt}} \qquad E_f = \frac{p_{12}^2}{2\mu_{12}} + \frac{P_{12}^2}{2M_{12}}$

integration over p_{12} with $d^3p_{12} = p_{12}^2 dp_{12} d\Omega_{12} \qquad \partial E_f/\partial p_{12} = p_{12}/\mu_{12}$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega_{12}} = \frac{2\pi}{\hbar}\frac{\mu_{pt}\mu_{12}}{(2\pi\hbar)^3}\frac{p_{12}}{p_{pt}}\frac{1}{N_{pt}}\sum_{i=(pt)}\sum_{f=(12)}|T_{fi}|^2$$

Reaction Theory - Cross Sections III

total cross sections

- two-body reaction

$$\sigma(p+t \to e_1+e_2) = \frac{2\pi}{\hbar} \frac{\mu_{pt}\mu_{12}}{(2\pi\hbar)^3} \frac{p_{12}}{p_{pt}} \frac{1}{N_{pt}} \sum_{i=(pt)} \sum_{f=(12)} \int |T_{fi}|^2 \, d\Omega_{12}$$

- inverse two-body reaction

$$\sigma(e_1+e_2 \to p+t) = \frac{2\pi}{\hbar} \frac{\mu_{12}\mu_{pt}}{(2\pi\hbar)^3} \frac{p_{pt}}{p_{12}} \frac{1}{N_{12}} \sum_{i=(12)} \sum_{f=(pt)} \int |T_{fi}|^2 \, d\Omega_{pt}$$

- time-reversal symmetry $\Rightarrow$ $|T_{fi}|^2 = |T_{if}|^2$
- ratio of total cross sections (integration over Ω_{12} and Ω_{pt} equivalent)

$\Rightarrow$ theorem of detailed balance $$\frac{\sigma(p+t \to e_1+e_2)}{\sigma(e_1+e_2 \to p+t)} = \frac{p_{12}^2}{p_{pt}^2} \frac{N_{12}}{N_{pt}}$$

Reaction Theory - Potential Scattering

basic approaches

- time-dependent description

  solve time-dependent Schrödinger equation with potential $V(\vec{r},t)$

  $$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left[-\frac{\hbar^2}{2\mu}\Delta + V(\vec{r},t)\right]\psi(\vec{r},t)$$ for t from $-\infty$ to ∞

  initial state: normalized wave package $\psi(\vec{r},t)$ for projectile nucleus

  final state: project on particular plane-wave state $\phi_{\vec{k}_f}(\vec{r},t) = \exp\left[i(\vec{k}_f \cdot \vec{r} - \omega t)\right]$

  $\Rightarrow$ transition amplitude $$a(\vec{k}_f) = \langle \phi_{\vec{k}_f} | \psi \rangle$$

- fixed-energy description

  solve time-independent Schrödinger equation with potential $V(r)$

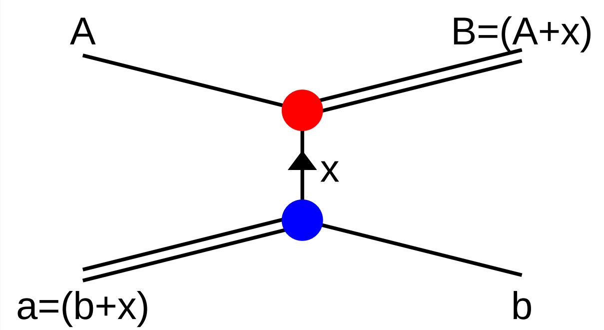
  $$E\psi(\vec{r}) = \left[-\frac{\hbar^2}{2\mu}\Delta + V(\vec{r})\right]\psi(\vec{r})$$ and energy $E = \frac{\hbar^2 k^2}{2\mu}$

  with boundary conditions for $\psi(\vec{r})$: plane wave + outgoing spherical waves

  $\Rightarrow$ S-matrix or T-matrix

Reaction Theory - T Matrix I

transfer reaction $A + a \to B + b$



- initial state
  - Hamiltonian $H_{Aa} = H_A + H_a + T_{Aa} + V_{Aa}$
  - wavefunctions $H_A \phi_A = E_A \phi_A \quad H_a \phi_a = E_a \phi_a$
- final state
  - Hamiltonian $H_{Bb} = H_B + H_b + T_{Bb} + V_{Bb}$
  - wavefunctions $H_B \phi_B = E_B \phi_B \quad H_b \phi_b = E_b \phi_b$
- solve Schrödinger equation $H\Psi_{Aa}^{(+)} = E\Psi_{Aa}^{(+)}$ with $H = H_{Aa} = H_{Bb}$

  and boundary condition $(r_{ij} \to \infty)$ for exact wave function

$$\Psi_{Aa}^{(+)} \to \phi_A \phi_a \exp\left(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}\right) + \sum_{(Bb)} f_{(Aa)(Bb)} \phi_B \phi_b \frac{\exp(ik_{Bb} r_{Bb})}{r_{Bb}}$$

  with reaction amplitudes $f_{(Aa)(Bb)}$

$\Rightarrow$ T-matrix element $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \exp\left(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}\right) | V_{Bb} | \Psi_{Aa}^{(+)} \rangle$

Reaction Theory - T Matrix II

transfer reaction $A + a \rightarrow B + b$

- exact expressions for T-matrix elements:
  - post form: $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \exp\left(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}\right) | V_{Bb} | \Psi^{(+)}_{Aa} \rangle$
  - prior form: $T_{(Bb)(Aa)} = \langle \Psi^{(-)}_{Bb} | V_{Aa} | \phi_A \phi_A \exp\left(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}\right) \rangle$
- plane-wave Born approximation (PWBA)
  - post form: $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \exp\left(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}\right) | V_{Bb} | \phi_A \phi_A \exp\left(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}\right) \rangle$
  - prior form: $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \exp\left(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}\right) | V_{Aa} | \phi_A \phi_A \exp\left(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}\right) \rangle$
- above expressions often not very useful in practical calculations

Reaction Theory - T Matrix III

transfer reaction $A + a \to B + b$

- introduce optical potentials U_{ij} $(ij = Aa, Bb)$

  and distorted waves $\chi_{ij}^{(\pm)}$ with $(T_{ij} + U_{ij})\,\chi_{ij}^{(\pm)} = E_{ij}\chi_{ij}^{(\pm)}$

- apply Gell-Mann–Goldberger relation (Phys. Rev. 91 (1953) 398) $\Rightarrow$

  ◦ post form: $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb} - U_{Bb} | \Psi_{Aa}^{(+)} \rangle$ exact!

  ◦ prior form: $T_{(Bb)(Aa)} = \langle \Psi_{Bb}^{(-)} | V_{Aa} - U_{Aa} | \phi_A \phi_A \chi_{Aa}^{(+)} \rangle$ exact!

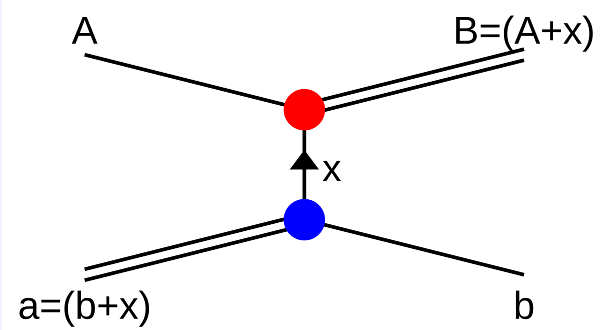
- distorted-wave Born approximation (DWBA)

  ◦ post form: $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb} - U_{Bb} | \phi_A \phi_a \chi_{Aa}^{(+)} \rangle$

  ◦ prior form: $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Aa} - U_{Aa} | \phi_A \phi_a \chi_{Aa}^{(+)} \rangle$

Reaction Theory - Spectroscopic Factors

transfer reaction $A + a \to B + b$



- overlap functions $\hat{=}$ wave function of transferred particle

$$\Phi^a_{bx} = \langle \phi_b | \phi_a \rangle \qquad \Phi^B_{Ax} = \langle \phi_A | \phi_B \rangle$$

- approximation with spectroscopic amplitudes and single-particle wave functions

$$\Phi^a_{bx} \approx \mathcal{A}^a_{bx} \varphi^a_{bx}(\vec{r}_{bx}) \phi_x \quad \Phi^B_{Ax} \approx \mathcal{A}^B_{Ax} \varphi^B_{Ax}(\vec{r}_{Ax}) \phi_x \quad \langle \varphi^a_{bx} | \varphi^a_{bx} \rangle = \langle \varphi^A_{Bx} | \varphi^A_{Bx} \rangle = 1$$

- spectroscopic factors $\mathcal{S}^a_{bx} = \langle \Phi^a_{bx} | \Phi^a_{bx} \rangle \approx \left| \mathcal{A}^a_{bx} \right|^2 \qquad \mathcal{S}^B_{Ax} = \langle \Phi^B_{Ax} | \Phi^B_{Ax} \rangle \approx \left| \mathcal{A}^B_{Ax} \right|^2$

- T-matrix elements in DWBA
  - post form: $T_{(Bb)(Aa)} = \langle \Phi^B_{Ax} \chi^{(-)}_{Bb} | V_{Bb} - U_{Bb} | \Phi^a_{bx} \chi^{(+)}_{Aa} \rangle$
  - prior form: $T_{(Bb)(Aa)} = \langle \Phi^B_{Ax} \chi^{(-)}_{Bb} | V_{Aa} - U_{Aa} | \Phi^a_{bx} \chi^{(+)}_{Aa} \rangle$

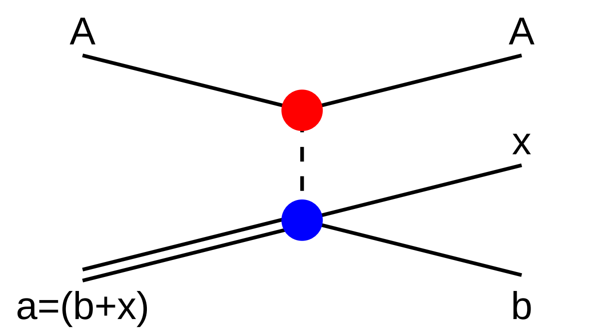
- cross sections $d\sigma \propto \left| T_{(Bb)(Aa)} \right|^2 \Rightarrow d\sigma \approx \mathcal{S}^a_{bx} \mathcal{S}^B_{Ax} d\sigma_{\text{single particle}}$

Reaction Theory - T Matrices for Indirect Methods

- **Coulomb Dissociation:** direct breakup reaction

  prior form $T_{(Bb)(Aa)} = \langle \Psi^{(-)}_{Bb} | V_{Aa} - U_{Aa} | \phi_A \phi_A \chi^{(+)}_{Aa} \rangle$

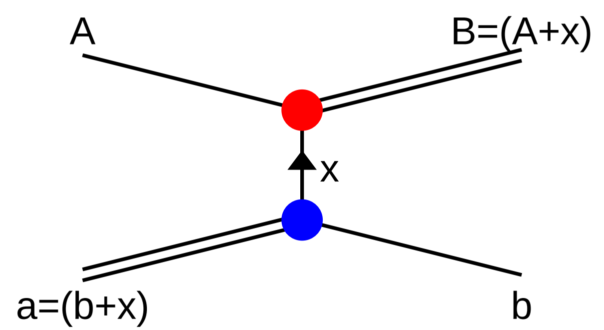
  with bound state wave function ϕ_a



- **ANC Method:** transfer reaction to bound state

  post form $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi^{(-)}_{Bb} | V_{Bb} - U_{Bb} | \Psi^{(+)}_{Aa} \rangle$

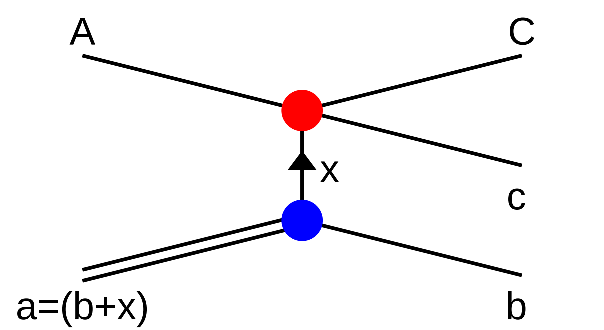
  with bound state wave function ϕ_B



- **Trojan-Horse Method:** transfer reaction to continuum

  post form $T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi^{(-)}_{Bb} | V_{Bb} - U_{Bb} | \Psi^{(+)}_{Aa} \rangle$

  with scattering wave function $\phi_B = \Psi^{(-)}_{Cc}$



Reaction Theory - Eikonal Approximation I

scattering of two particles $A + a$

- partial-wave expansion
  exact for all scattering angles, but sum over many orbital angular momenta l
- eikonal approximation:
  high-energy approximation for small scattering angles

- ansatz for scattering wave function with $\vec{r}_{Aa} = \vec{r}_A - \vec{r}_a = \vec{b} + z\vec{e}_z, \quad \vec{b} \perp \vec{e}_z$

$$\chi_{Aa}^{(+)}(\vec{k}_i) = \exp\left(i\vec{k}_i \cdot \vec{r}_{Aa}\right) \exp\left[iS_{Aa}^{(+)}(z,\vec{b})\right]$$

$$\chi_{Aa}^{(-)}(\vec{k}_f) = \exp\left(i\vec{k}_f \cdot \vec{r}_{Aa}\right) \exp\left[-iS_{Aa}^{(-)*}(z,\vec{b})\right]$$

- Schrödinger equation with optical potential U_{Aa}
  neglect derivatives with respect to x, y $\Rightarrow$ phase functions

$$S_{Aa}^{(+)}(z,\vec{b}) = -\frac{1}{\hbar v_{Aa}} \int_{-\infty}^{z} dz' \, U_{Aa}(\vec{r}\,'_{Aa}) \qquad S_{Aa}^{(-)}(z,\vec{b}) = -\frac{1}{\hbar v_{Aa}} \int_{z}^{\infty} dz' \, U_{Aa}(\vec{r}\,'_{Aa})$$

Reaction Theory - Eikonal Approximation II

scattering of two particles $A + a$

- form factor ($\Rightarrow$ T-matrix element)

$$F = \langle \chi_{Aa}^{(-)}(\vec{k}_f)|V|\chi_{Aa}^{(+)}(\vec{k}_i)\rangle = \int d^3r_{Aa}\, \exp\left(i\vec{q}\cdot\vec{r}_{Aa}\right) \exp\left[iS_{Aa}(\vec{b})\right] V$$

with momentum transfer $\vec{q} = \vec{k}_i - \vec{k}_f$

and phase function $S_{Aa}(\vec{b}) = -\frac{1}{\hbar v_{Aa}} \int_{-\infty}^{\infty} dz'\, U_{Aa}(\vec{r}'_{Aa})$

- small-angle scattering: $\vec{q}\cdot\vec{r}_{Aa} \approx \vec{q}\cdot\vec{b}$ with $q \approx 2\sqrt{k_i k_f}\sin\frac{\vartheta_{Aa}}{2}$

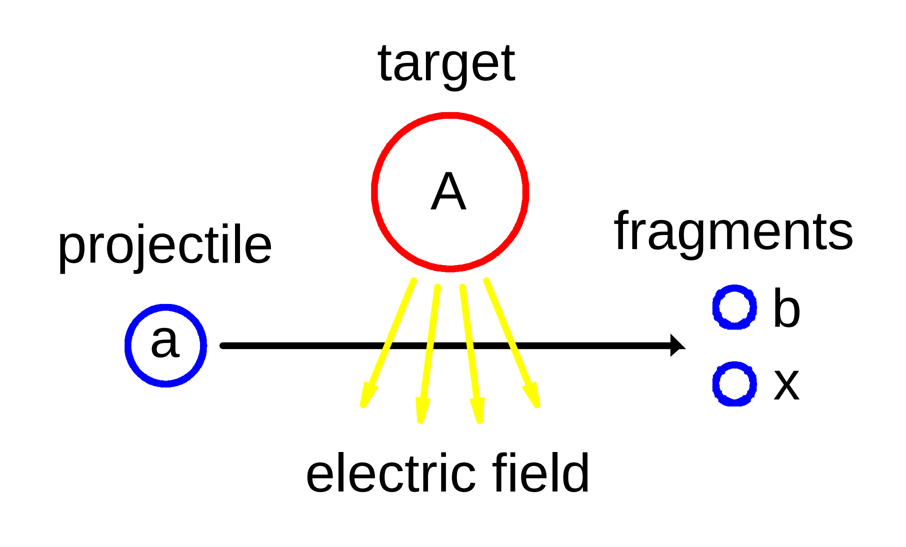
$\Rightarrow$ form factor $F = \int d^2b\, \exp\left(i\vec{q}\cdot\vec{b}\right) \exp\left[iS_{Aa}(\vec{b})\right] \int_{-\infty}^{\infty} dz\, V$

- elastic scattering: $V = U_{Aa}$, $k_i = k_f \Rightarrow$ T-matrix element

$$T_{fi} = \langle \exp\left(i\vec{k}_f\cdot\vec{r}_{Aa}\right)|U_{Aa}|\chi_{Aa}^{(+)}(\vec{k}_i)\rangle = i\hbar v_{Aa} \int d^2b\, \exp\left(i\vec{q}\cdot\vec{b}\right) \exp\left[iS_{Aa}(\vec{b})\right]$$

(spherical symmetry $\Rightarrow$ one-dimensional integral)

Coulomb Dissociation - Idea



radiative capture $b(x,\gamma)a$

detailed balance $\Updownarrow$

photo absorption $a(\gamma,x)b$

equivalent photons in Coulomb field of target A $\Updownarrow$

Coulomb dissociation $A(a,bx)A$

(G. Baur, H. Rebel, C. Bertulani, Nucl. Phys. A 458 (1986) 188)

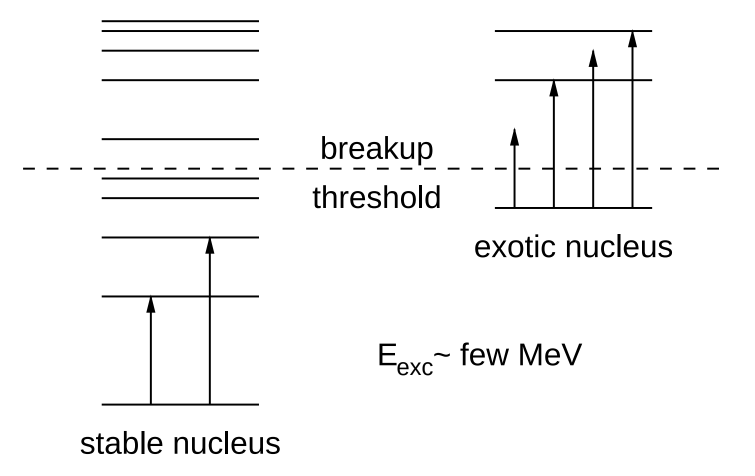
correspondence

(Fermi 1924, Weizsäcker-Williams 1932)

time-dependent electromagnetic field
of highly-charged nucleus A
during scattering of projectile a

$\Updownarrow$

spectrum of (virtual, equivalent) photons



only ground state transitions !

Coulomb Dissociation - Theory

Coulomb dissociation reaction: $a + A \to b + x + A$

with three-body final state in the continuum

$\Rightarrow$ only approximate theoretical treatment

- **semiclassical methods**

– classical description of projectile-target relative motion
(valid for heavy targets if $\eta_{Aa} = Z_A Z_a e^2/(\hbar v) \gg 1$ with beam velocity v)

– time-dependent perturbation $V(t)$ of projectile system

– time-dependent perturbation theory

$\Rightarrow$ excitation amplitude a_{fi}

- **quantal methods**

– valid for all projectile/target combinations and all beam energies

– time-independent scattering theory

$\Rightarrow$ T-matrix element T_{fi}

Coulomb Dissociation - Semiclassical Theory I

first-order semiclassical approximation for reaction $A(a, bx)A$

- classical description of projectile-target relative motion $\Rightarrow$ $\vec{R}_A(t)$

  valid for heavy targets if $\eta_{Aa} = Z_A Z_a e^2/(\hbar v_{Aa}) \gg 1$ with beam velocity v

- time-dependent perturbation of projectile system (magnetic interaction neglected)

$$V(t) = \frac{Z_b Z_A e^2}{|\vec{r}_b - \vec{R}_A(t)|} + \frac{Z_x Z_A e^2}{|\vec{r}_x - \vec{R}_A(t)|} - \frac{Z_a Z_A e^2}{|\vec{r}_a - \vec{R}_A(t)|}$$

- excitation amplitude in first-order time-dependent perturbation theory

$$a_{fi} = \frac{1}{i\hbar} \int dt \; \exp(i\omega t) \langle f | V(t) | i \rangle \qquad \begin{aligned} |i\rangle &= |J_a M_a\rangle \\ |f\rangle &= |\vec{k}_{bx} J_{bx} M_{bx}\rangle \end{aligned}$$

  with excitation energy $\hbar\omega = E_f - E_i = E_\gamma = E_{bx} + S_{bx} = \frac{\hbar^2 k_{bx}^2}{2\mu_{bx}} + S_{bx}$

Coulomb Dissociation - Semiclassical Theory II

- excitation probability $$P_{fi} = \frac{1}{2J_a + 1} \sum_{M_a} \sum_{M_{bx}} \left| a_{fi} \right|^2 \varrho$$

  with density of final states $$\varrho = \frac{p_{bx}^2}{(2\pi\hbar)^3} \frac{dp_{bx}}{dE_{bx}} = \frac{\mu_{bx} k_{bx}}{(2\pi)^3 \hbar^2}$$

- Coulomb breakup cross section $$\frac{d^3\sigma}{dE_{bx} d\Omega_{bx} d\Omega_{aA}} = P_{fi} \frac{d\sigma_R}{d\Omega_{aA}}$$

  with Rutherford cross section $\frac{d\sigma_R}{d\Omega_{aA}}$ for elastic $a + A$ scattering

- three-body final state $\Rightarrow$
  most general cross section depends on on 5 quantities
  (one energy, four angles)

Coulomb Dissociation - Semiclassical Theory III

- multipole expansion of Coulomb potential in far-field approximation ($r_{bx} < r_{aA}$)

$$V(t) = \frac{Z_b Z_A e^2}{|\vec{r}_b - \vec{r}_A|} + \frac{Z_x Z_A e^2}{|\vec{r}_x - \vec{r}_A|} - \frac{Z_a Z_A e^2}{|\vec{r}_a - \vec{r}_A|} \approx 4\pi Z_A e \sum_{\lambda\mu} \frac{Z_{\text{eff}}^{(\lambda)} e}{2\lambda+1} \frac{r_{bx}^{\lambda}}{r_{Aa}^{\lambda+1}} Y_{\lambda\mu}(\hat{r}_{bx}) Y_{\lambda\mu}^{*}(\hat{r}_{Aa})$$

with effective charge numbers $Z_{\text{eff}}^{(\lambda)} = Z_b \left(\frac{m_x}{m_b+m_x}\right)^{\lambda} + Z_x \left(-\frac{m_b}{m_b+m_x}\right)^{\lambda}$

and relative coordinates $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, $\vec{r}_{Aa} = \vec{r}_A - \vec{r}_a$

- factorization of excitation amplitude

$$a_{fi} = \frac{Z_A e}{i\hbar} \sum_{\lambda\mu} \frac{4\pi}{2\lambda+1} \langle f | \underbrace{Z_{\text{eff}}^{(\lambda)} e r_{bx}^{\lambda} Y_{\lambda\mu}(\hat{r}_{bx})} | i \rangle \int dt \, \exp(i\omega t) \frac{Y_{\lambda\mu}^{*}(\hat{r}_{Aa})}{r_{Aa}^{\lambda+1}}$$

$\mathcal{M}(E\lambda\mu)$ electric multipole transition operator

$\Rightarrow$ transition matrix element $\times$ semiclassical Coulomb integral

$a_{fi} \propto Z_A \Rightarrow$ use highly charged target nucleus A

Coulomb Dissociation - Photonuclear Transitions

- introduction of reduced matrix elements (Wigner-Eckart theorem)

$$\langle f | Z_{\text{eff}}^{(\lambda)} e r_{bx}^{\lambda} Y_{\lambda\mu}(\hat{r}_{bx}) | i \rangle \qquad |i\rangle = |J_a M_a\rangle \qquad |f\rangle = |\vec{k}_{bx} J_{bx} M_{bx}\rangle$$

$$= \langle \vec{k}_{bx} J_{bx} M_{bx} | \mathcal{M}(E\lambda\mu) | J_a M_a \rangle = (J_a \, M_a \, \lambda \, \mu | J_{bx} \, M_{bx}) \langle \vec{k}_{bx} J_{bx} || \mathcal{M}(E\lambda) || J_a \rangle$$

- definition of reduced transition probability for excitation to continuum

$$\frac{dB(E\lambda)}{dE} = \frac{2J_{bx}+1}{2J_a+1} \left| \langle k_{bx} J_{bx} || \mathcal{M}(E\lambda) || J_a \rangle \right|^2 \frac{\mu_{bx} k_{bx}}{(2\pi)^3 \hbar^2}$$

- photo absorption cross section (for electric transition of multipolarity λ)

$$\sigma_{E\lambda}(a+\gamma \to b+x) = \frac{\lambda+1}{\lambda} \frac{(2\pi)^3}{[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda-1} \frac{dB(E\lambda)}{dE}$$

$\Rightarrow$ rewrite Coulomb dissociation cross section

Coulomb Dissociation - Semiclassical Theory IV

Coulomb dissociation cross section
(with angular integration over relative momentum between fragments)

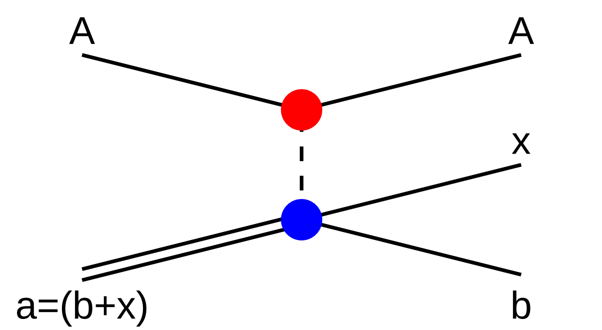
$$\Rightarrow \quad \frac{d^2\sigma}{dE_{bx}d\Omega_{aA}} = \frac{1}{E_\gamma}\sum_{\pi\lambda}\sigma_{\pi\lambda}(a+\gamma \to b+x)\,\frac{dn_{\pi\lambda}}{d\Omega_{aA}} \qquad \pi = E, M \qquad \lambda = 1, 2, \ldots$$

- photo absorption cross section $\sigma_{\pi\lambda}(a+\gamma \to b+x)$
- virtual photon numbers $\frac{dn_{\pi\lambda}}{d\Omega_{aA}}$ that depend on kinematics:
  - ○ scattering angle ϑ_{aA}/impact parameter b
  - ○ projectile velocity v
  - ○ excitation energy $E_\gamma = \hbar\omega$

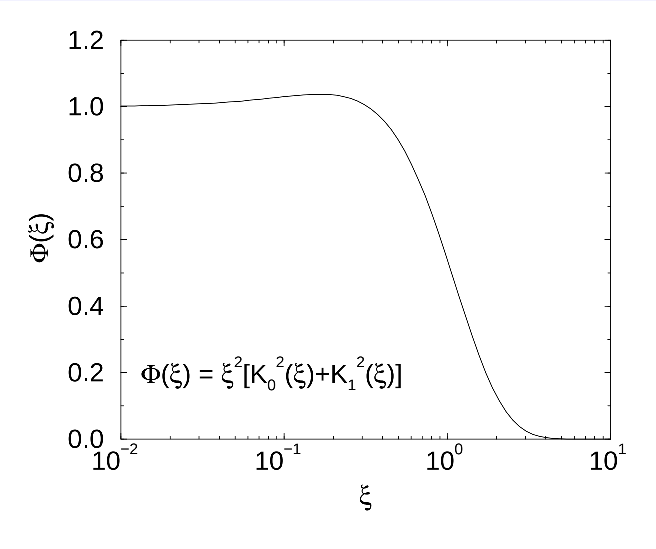
  calculation in
  - ○ non-relativistic approximation with Coulomb (hyperbolic) scattering trajectories
  - ○ relativistic approximation with straight-line trajectories

  $\Rightarrow$ E2 enhancement $\frac{dn_{E2}}{d\Omega_{aA}}\Big/\frac{dn_{E1}}{d\Omega_{aA}} \approx \frac{4\hbar^2c^2}{E_\gamma^2 b^2}$ M1 suppression $\frac{dn_{M1}}{d\Omega_{aA}}\Big/\frac{dn_{E1}}{d\Omega_{aA}} \approx \frac{v^2}{c^2}$

Coulomb Dissociation - Characteristic Parameters



virtual photon spectrum (E1)



(Fermi 1924, Weizsäcker-Williams 1932)

- **adiabaticity parameter**

$$\xi = \frac{\omega b}{\gamma v} = \frac{\text{duration of scattering process}}{\text{excitation period}}$$

$\xi = 0$: sudden excitation
$\xi \gg 1$: adiabatic excitation
$\xi \approx 1 \Rightarrow E_{\text{exc}}^{\text{max}} \approx \gamma v \hbar / b$

- **strength parameter**

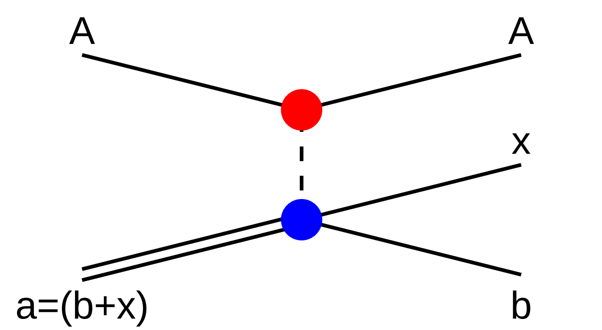
$$\chi = \frac{Z_A e \langle f || \mathcal{M}(\pi\lambda) || i \rangle}{\hbar v b^{\lambda}}$$

$Z_A e$ target charge
$\mathcal{M}(\pi\lambda)$ multipole operator

χ small $\Rightarrow$ first-order perturbation theory sufficient

χ large $\Rightarrow$ higher-order effects

Coulomb Dissociation - Relation of Cross Sections



- Coulomb dissociation cross section

$$\frac{d^2\sigma}{dE_{bx}d\Omega_{Aa}} = \frac{1}{E_\gamma}\sum_{\pi\lambda}\sigma_{\pi\lambda}(a+\gamma\rightarrow b+x)\frac{dn_{\pi\lambda}}{d\Omega_{Aa}}$$

- theorem of detailed balance

$$\sigma_{\pi\lambda}(a+\gamma\rightarrow b+x) = \frac{(2J_b+1)(2J_x+1)}{2(2J_a+1)}\,\frac{k_{bx}^2}{k_\gamma^2}\,\sigma_{\pi\lambda}(b+x\rightarrow a+\gamma)$$

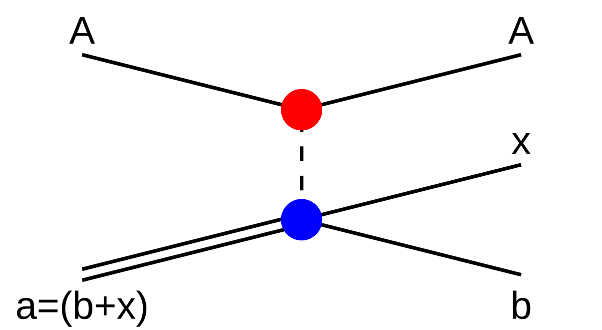
with photo absorpton and radiative capture cross sections

- phase space factor $\frac{k_{bx}^2}{k_\gamma^2} = \frac{2\mu_{bx}c^2E_{bx}}{(E_{bx}+S_{bx})^2} \gg 1$ for not too small E_{bx}

- virtual photon numbers $\frac{dn_{\pi\lambda}}{d\Omega_{Aa}} \gg 1$ for large Z_A and for not too high E_{bx} $(\hat{=}\,\xi)$

$\Rightarrow$ **large Coulomb dissociation cross sections**

Coulomb Dissociation - Quantal Theory I



- prior-form distorted-wave Born approximation (DWBA)

$$T_{fi} = \langle \chi^{(-)}_{A(bx)} \phi_A \Psi^{(-)}_{bx} | V_{Aa} - U_{Aa} | \phi_A \phi_a \chi^{(+)}_{Aa} \rangle$$

- neglection of nuclear interaction in V_{Aa} and U_{Aa}

- multipole expansion of Coulomb potential in far-field approximation ($r_{bx} < r_{Aa}$)

$$V^{(i)}_{Aa} - U_{Aa} = \frac{Z_A Z_b e^2}{|\vec{r}_b - \vec{r}_A|} + \frac{Z_A Z_x e^2}{|\vec{r}_x - \vec{r}_A|} - \frac{Z_A Z_a e^2}{|\vec{r}_a - \vec{r}_A|} \approx 4\pi Z_A e \sum_{\lambda\mu} \frac{Z^{(\lambda)}_{\text{eff}} e}{2\lambda+1} \frac{r^{\lambda}_{bx}}{r^{\lambda+1}_{Aa}} Y_{\lambda\mu}(\hat{r}_{bx}) Y^*_{\lambda\mu}(\hat{r}_{Aa})$$

with effective charge numbers $Z^{(\lambda)}_{\text{eff}} = Z_b \left(\frac{m_x}{m_b+m_x}\right)^{\lambda} + Z_x \left(-\frac{m_b}{m_b+m_x}\right)^{\lambda}$

and relative coordinates $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, $\vec{r}_{Aa} = \vec{r}_A - \vec{r}_a$

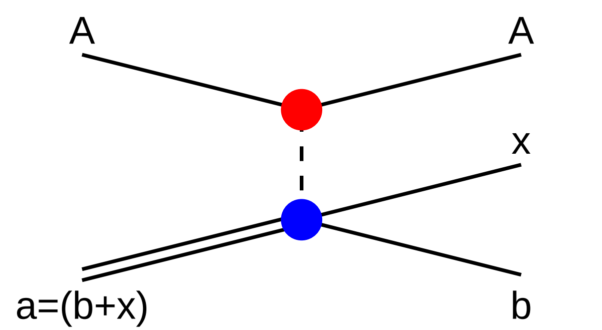
$\Rightarrow$ factorization of T-matrix element

$$T_{fi} \approx Z_A e \sum_{\lambda\mu} \frac{4\pi}{2\lambda+1} \langle \Psi^{(-)}_{bx} | \underbrace{Z^{(\lambda)}_{\text{eff}} e r^{\lambda}_{bx} Y_{\lambda\mu}(\hat{r}_{bx})}_{\mathcal{M}(E\lambda\mu)} | \phi_a \rangle \langle \chi^{(-)}_{A(bx)} | r^{-\lambda-1}_{Aa} Y^*_{\lambda\mu}(\hat{r}_{Aa}) | \chi^{(+)}_{Aa} \rangle$$

$\mathcal{M}(E\lambda\mu)$ electric multipole transition operator

$\Rightarrow$ transition matrix element $\times$ quantal Coulomb integral

Coulomb Dissociation - Quantal Theory II



- quantal Coulomb integral

$$\langle \chi^{(-)}_{A(bx)} | r_{Aa}^{-\lambda-1} Y^{*}_{\lambda\mu}(\hat{r}_{Aa}) | \chi^{(+)}_{Aa} \rangle$$

with Coulomb distorted waves $\chi^{(+)}_{Aa}$ and $\chi^{(-)}_{A(bx)}$

- exact calculation with partial-wave expansion

$$\chi^{(+)}_{Aa} = \frac{4\pi}{k_{Aa} r_{Aa}} \sum_{lm} e^{i\sigma_l} F_l(\eta_{Aa}, k_{Aa} r_{Aa}) i^l Y_{lm}(\hat{r}_{Aa}) Y^{*}_{lm}(\hat{k}_{Aa}) \qquad \chi^{(-)}_{A(bx)} = \dots$$

with regular Coulomb wave functions F_l and Coulomb phase shifts σ_l

$\Rightarrow$ numerically involved

- eikonal approximation with Coulomb shift function $S(b) = \exp\left[2i\eta_{Aa} \ln(k_{Aa} b)\right]$

$$\langle \chi^{(-)}_{A(bx)} | r_{Aa}^{-\lambda-1} Y^{*}_{\lambda\mu}(\hat{r}_{Aa}) | \chi^{(+)}_{Aa} \rangle = \int d^2 b \, \exp\left(i\vec{q} \cdot \vec{b}\right) \, \exp\left[iS(b)\right] \int dz \, r_{Aa}^{-\lambda-1} Y^{*}_{\lambda\mu}(\hat{r}_{Aa})$$

evaluation with method of steepest descent/saddle-point approximation

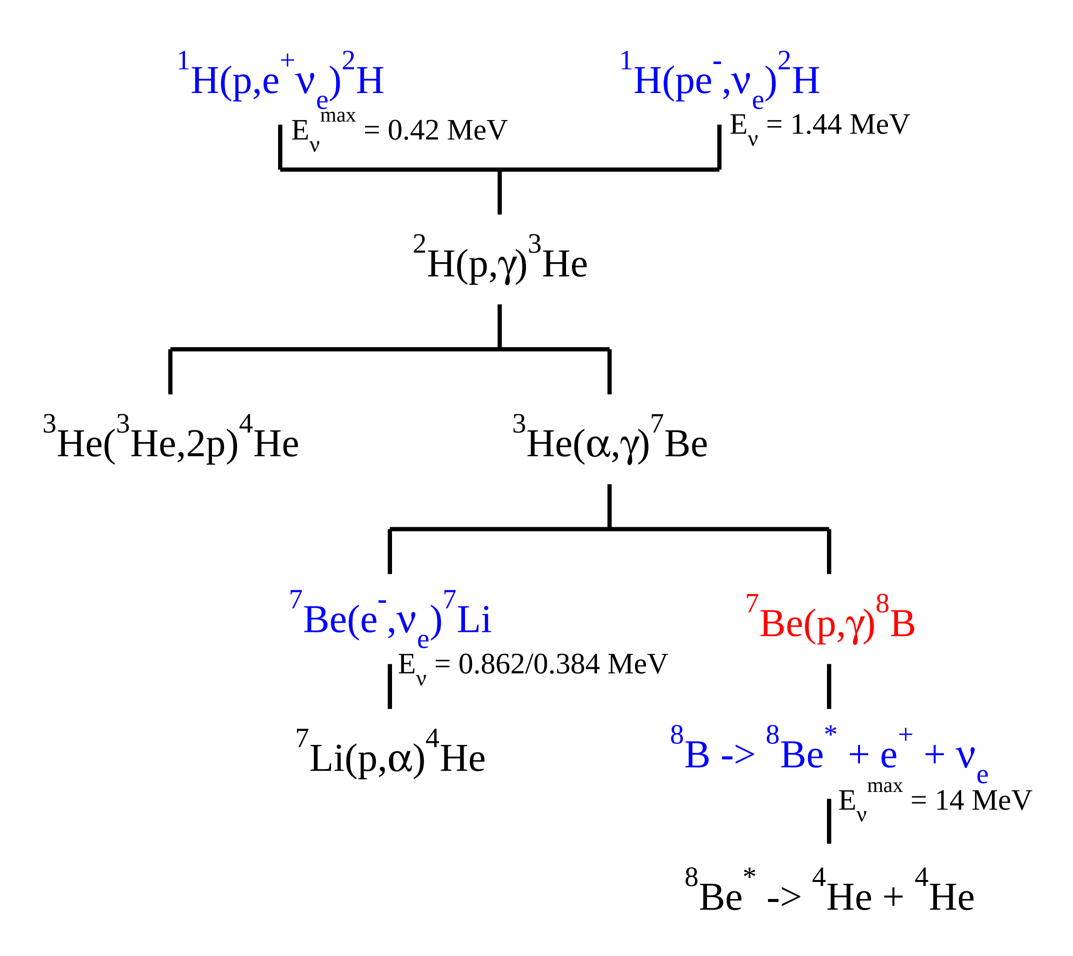
$\Rightarrow$ classical relations $q = \frac{2\eta_{Aa}}{b} = 2k_{Aa} \sin\frac{\vartheta_{Aa}}{2}, \quad \xi = \frac{\omega b}{\gamma v_{Aa}}$, semiclassical result

Coulomb Dissociation - Example: ^{8}B

^{8}B and solar neutrinos

- pp chains: main source of solar energy production
- flux of high-energy neutrinos proportional to synthesized ^{8}B
- precise knowledge of ^{7}Be(p,γ)^{8}B S factor $S_{17}(E)$ in Gamov window ($E \approx 20$ keV) required
- solar neutrino problem solved with neutrino oscillation
- more precise direct capture data available recently

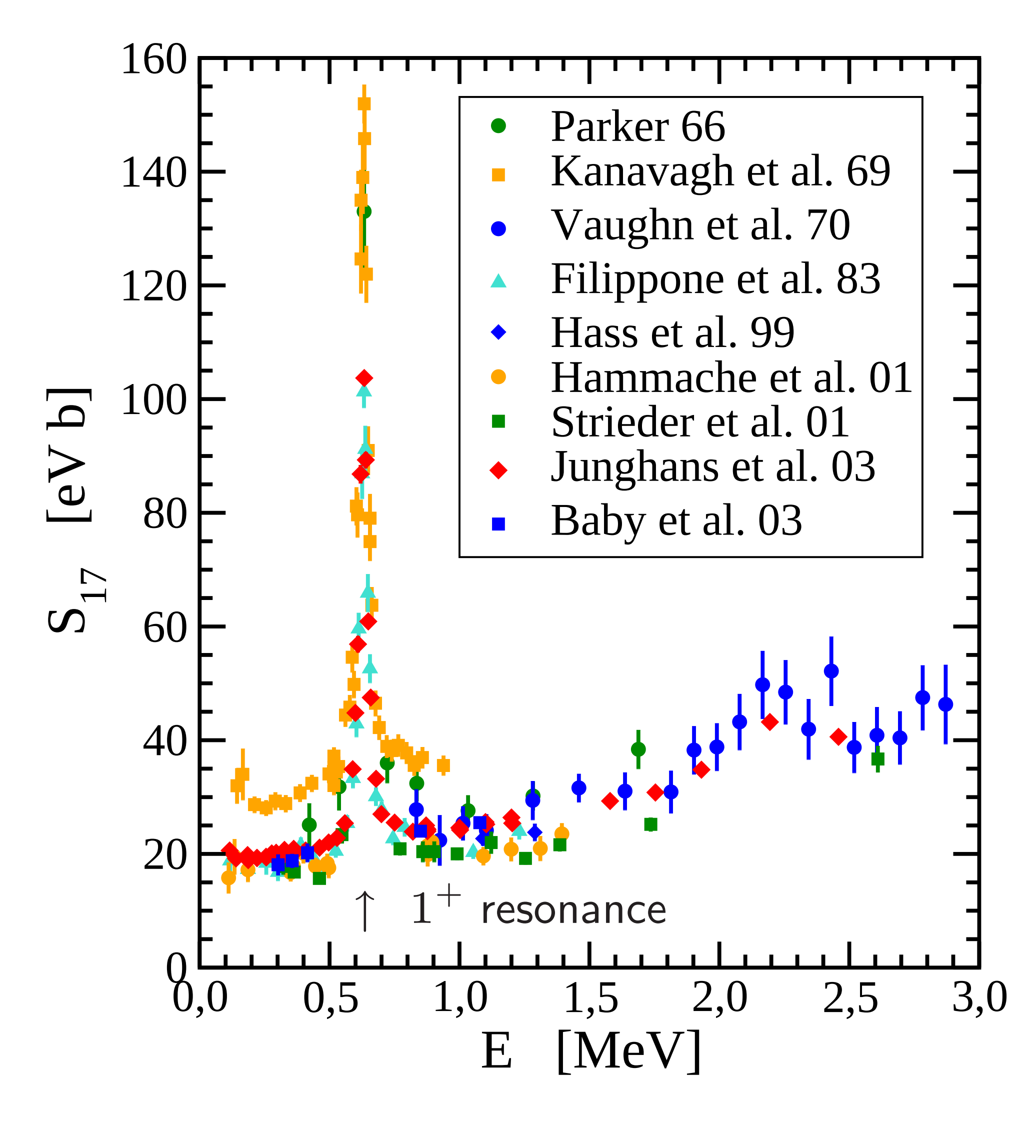
$\Rightarrow$ **test case** for Coulomb dissociation method



Coulomb Dissociation - Example: ^{7}Be(p,γ)^{8}B

direct experiments

- R.W. Kavanagh, Nucl. Phys. 15 (1960) 411
- P.D. Parker, Phys. Rev. 150 (1966) 851;
  Astrophys. J. 153 (1968) L85
- R.W. Kavanagh et al., Bull. Am. Phys. Soc. 14 (1969) 1209
- F.J. Vaughn et al., Phys. Rev. C 2 (1970) 1657
- C. Wiezoreck et al., Z. Phys. A 282 (1977) 121
- B.W. Filippone et al., Phys. Rev. Lett. 50 (1983) 412;
  Phys. Rev. C 28 (1983) 2222
- M. Hass et al., Phys. Lett. B 462 (1999) 237
- F. Hammache et al., Phys. Rev. Lett. 80 (1988) 928;
  Phys. Rev. Lett. 86 (2001) 3985
- F. Strieder et al., Nucl. Phys. A 696 (2001) 219
- A.R. Junghans et al., Phys. Rev. Lett. 88 (2002) 041101;
  Phys. Rev. C 68 (2003) 065803
- T. Baby et al., Phys. Rev. C 67 (2003) 065805

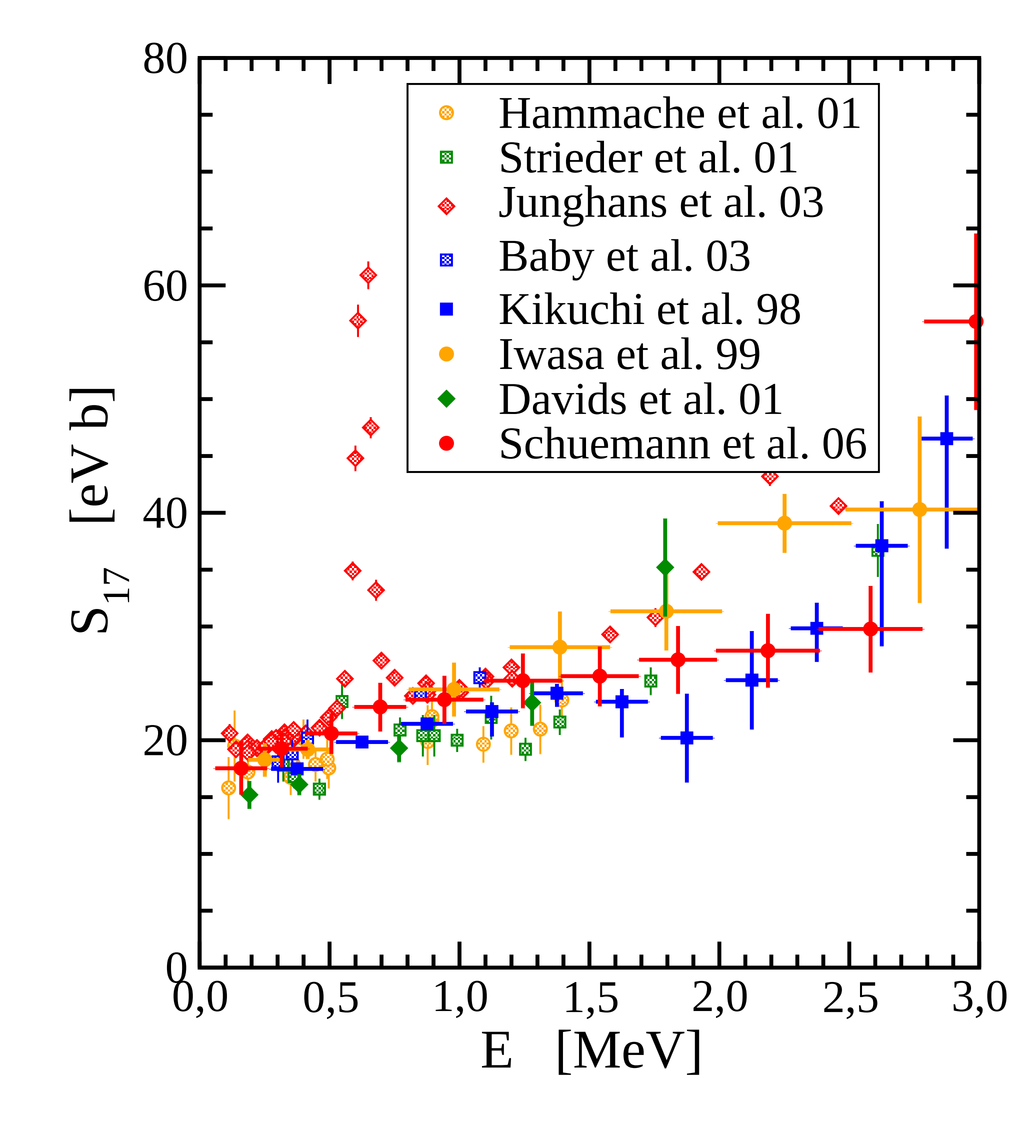


improvement in precision and consistency

Coulomb Dissociation - Example: ^{7}Be(p,γ)^{8}B

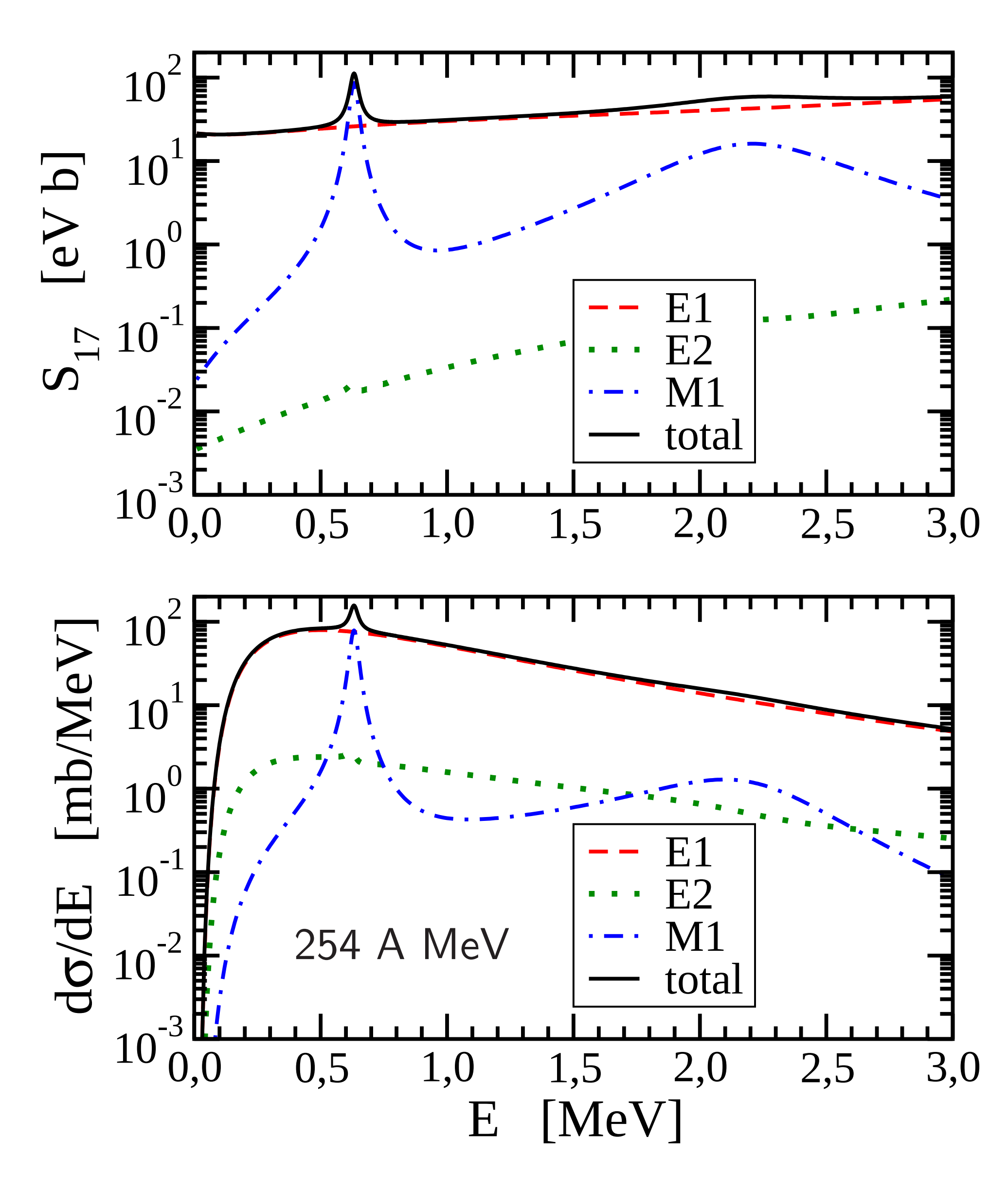
Coulomb dissociation experiments

- T. Motobayashi et al., PRL 73 (1994) 2680
  RIKEN-1, 46.5 A MeV, $\frac{d\sigma}{dE}$, $\frac{d\sigma}{d\Omega_8}$ in E bins
  $S_{17}(0) = 16.7 \pm 3.2$ eV b
- T. Kikuchi et al., EPJ A 3 (1998) 213
  RIKEN-2, 51.9 A MeV, $\frac{d\sigma}{dE}$
  $S_{17}(0) = 19.6 \pm 0.3(\text{stat}) \pm 1.6(\text{syst})$ eV b
- N. Iwasa et al., PRL 83 (1999) 2910
  GSI-1, 254 A MeV, $\frac{d\sigma}{dE}$, $\frac{d\sigma}{d\Omega_8}$ in E bins
  $S_{17}(0) = 20.6 \pm 1.2(\text{exp}) \pm 1.0(\text{theo})$ eV b
- B. Davids et al., PRC 63 (2001) 065806
  MSU, 44/81/83 A MeV, $\frac{d\sigma}{dp_l}$, $\frac{d\sigma}{dE}$
  $S_{17}(0) = 17.8^{+1.4}_{-1.2}$ eV b
- F. Schümann et al., PRC 73 (2006) 015806
  GSI-2, 254 A MeV, $\frac{d\sigma}{d\theta_{17}}$, $\frac{d\sigma}{d\Omega_8}$, $\frac{d\sigma}{dE}$, ...
  $S_{17}(0) = 20.6 \pm 0.8(\text{exp}) \pm 1.2(\text{theo})$ eV b
  most complete and precise indirect experiment



Coulomb Dissociation - Example: ^{8}B Model

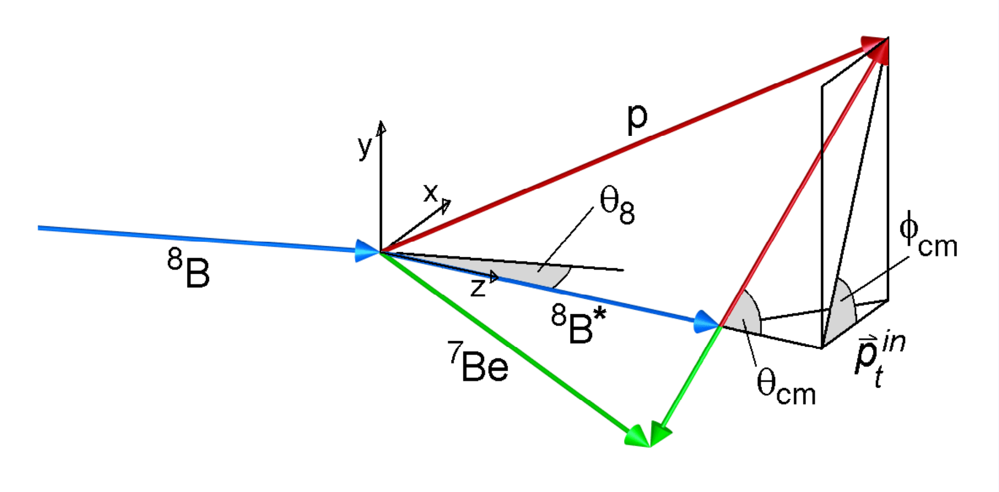
- ^{7}Be-p potential model
- depths of Woods-Saxon potentials adjusted to binding energy of 2^+ p-wave ground state (0.1375 MeV, halo system), 1^+ and 3^+ resonance energies, s-wave scattering lengths
- E1, E2 strength from model, M1 scaled to experimental strength
- Coulomb breakup in relativistic semiclassical approximation with quantal correction for diffraction
- nuclear breakup not included
- full triple differential cross section converted to event distribution, input for GEANT simulation

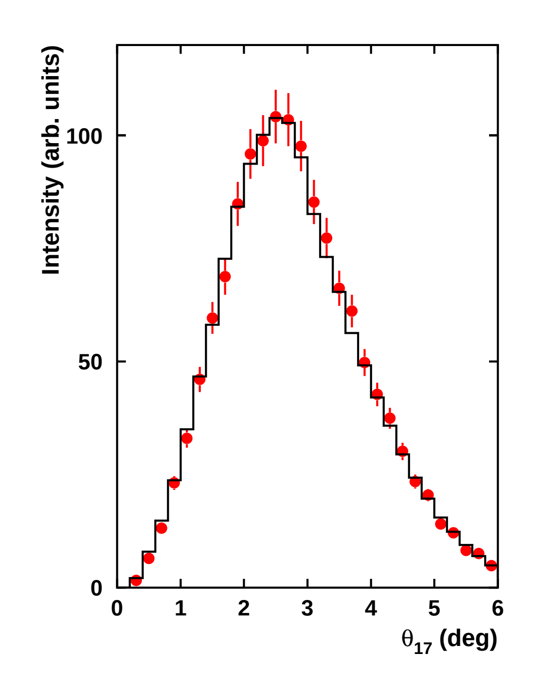


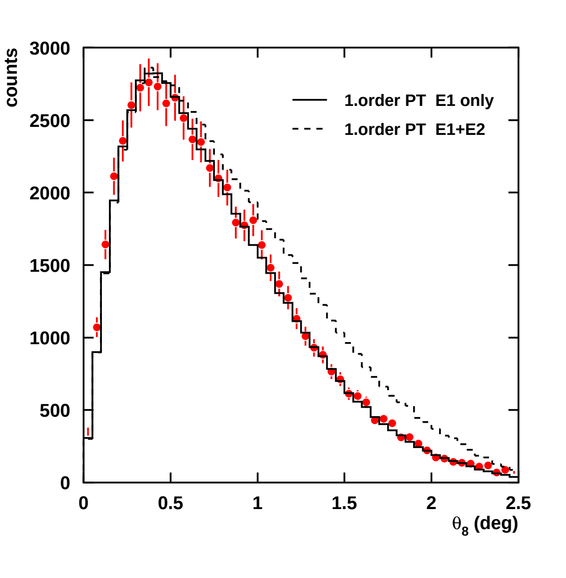
Coulomb Dissociation - Example: ^{8}B

- high-energy ^{8}B beam: 254 A MeV
  $\Rightarrow$ small higher-order effects
  $\Rightarrow$ small nuclear breakup (Re $V_{\mathrm{pot}} \approx 0$), absorption important ($\Leftrightarrow$ Im V_{opt})
- kinematically complete with high statistics and precision
  $\Rightarrow$ various angular and momentum distributions
  $\Rightarrow$ no sign for substantial E2 contribution or higher-order effects
- M1 contribution suppressed as compared to capture reaction, but observed
- model predictions close to experiment
  $\Rightarrow$ only small adjustment of model S factor required

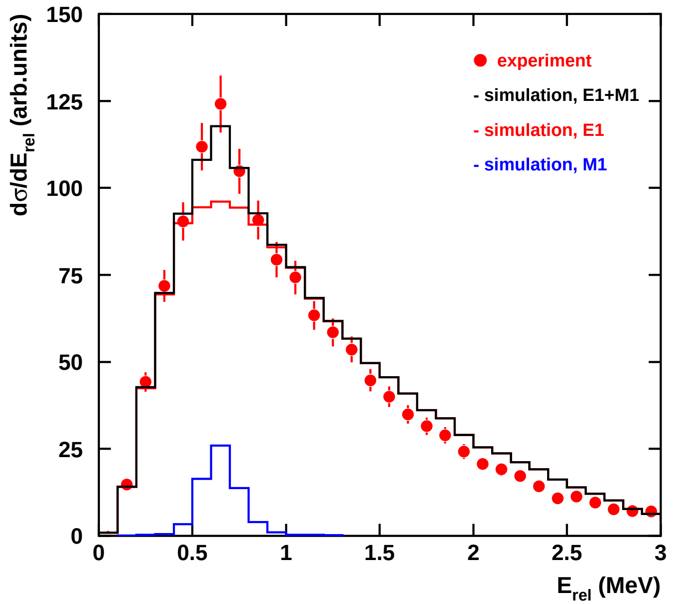
(F. Schümann et al., PRC 73 (2006) 015806)

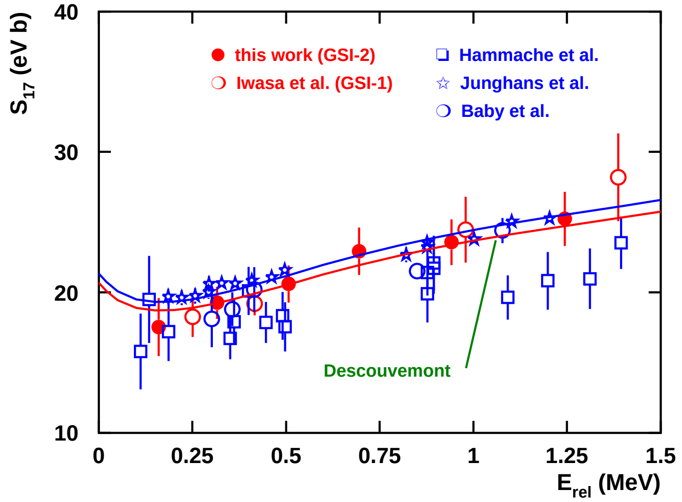






Coulomb Dissociation - Example: ^{8}B

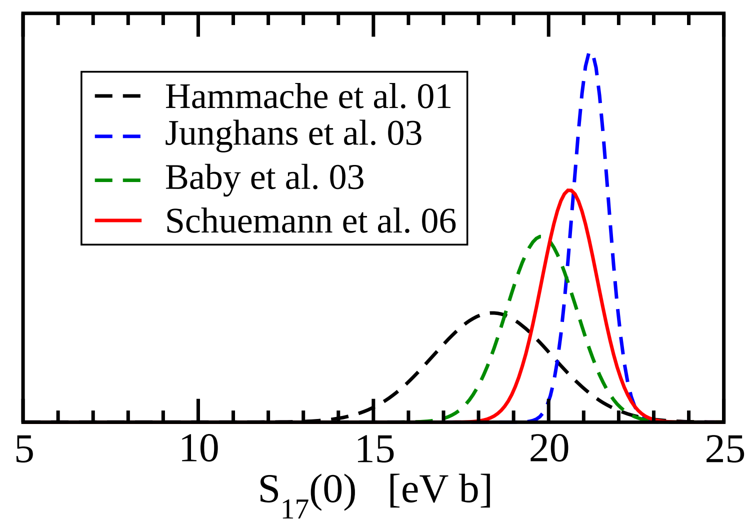




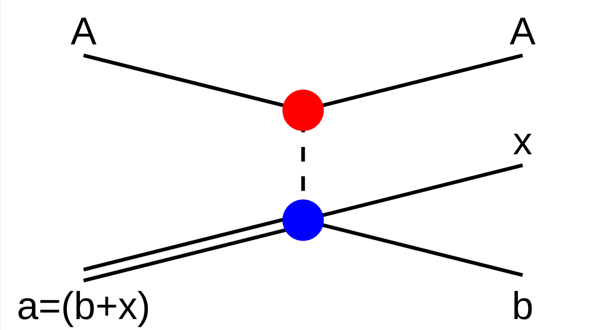
- difference experiment - simulation in $\frac{d\sigma}{dE}$
  $\Rightarrow$ adjustment of model $S_{17}(E)$
- extrapolation of S factor to zero energy
  with microscopic cluster model by
  P. Descouvement, PRC 70 (2004) 065802
  $\Rightarrow$ normalization factor: 0.837 ± 0.013

$\Rightarrow$ $S_{17}(0) = 20.6 \pm 0.8$ (stat) ± 1.2 (syst) eV b

consistent with most precise direct results



Coulomb Dissociation - Higher-Order Effects



interaction of fragments in final state
with target Coulomb field after breakup
⇒ "post-acceleration" $\hat{=}$ multi-photon exchange
⇒ Coulomb dissociation cross section
not proportional to photo absorption cross section

theoretical approaches

- semiclassical description
  - higher-order perturbation theory
  - full dynamical calculation (solving the time-dependent Schrödinger equation)
- quantal description
  - prior-form DWBA $T_{fi} = \langle \chi^{(-)}_{A(bx)} \phi_A \Psi^{(-)}_{bx} | V_{Aa} - U_{Aa} | \phi_A \phi_a \chi^{(+)}_{Aa} \rangle$

    first order in V_{Aa}, all orders in V_{bx}
  - post-form DWBA $T_{fi} = \langle \chi^{(-)}_{Ab} \chi^{(-)}_{Ax} \phi_A \phi_b \phi_x e^{i\vec{k}_{bx}\cdot\vec{r}_{bx}} | V_{bx} | \phi_A \phi_a \chi^{(+)}_{Aa} \rangle$

    first order in V_{bx}, all orders in V_{Ab} and V_{Ax}

    ⇒ factorization, Bremsstrahlung integrals

Wave Functions - General

relative motion of two-body system $a = b + x \, (= c + y = d + z = \ldots)$

- many-body wave function Ψ_a is solution of Schrödinger equation

  $$H\Psi_a = (T + V_a)\,\Psi_a = E\Psi_a$$

  with potential $V_a = V_{bx}^C + V_{bx}^N = V_{cy}^C + V_{cy}^N$ (Coulomb + nuclear interaction)
  and boundary condition for bound/scattering state

- for large distances $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, etc.:
  Coulomb interaction remains, short-range nuclear interaction vanishes
  $\Rightarrow$ universal asymptotic form of $\Psi_a \to \phi_b \phi_x \psi_{bx}(r_{bx}) + \ldots$
  for $r_{bx}, r_{cy}, \ldots \to \infty$ with relative wave functions $\psi_{bx}, \psi_{cy}, \ldots$

- exact solution: partial wave expansion

- approximate solution for high-energy and forward scattering: eikonal approximation

Wave Functions - Bound States

general form of asymptotics (without particle spins, $\alpha = (bx), (cy), \ldots$)

$$\psi_\alpha(m) \to \frac{1}{r_\alpha} \sum_l f_{\alpha l}(r_\alpha) Y_{lm}(\hat{r}_\alpha)$$ for $r_\alpha \to \infty$

with radial wave functions $f_{\alpha l}(r_\alpha) = C_\alpha^a(l) W_{-\eta_\alpha, l+1/2}(2 q_\alpha r_\alpha)$

and angular parts $Y_{lm}(\hat{r}_\alpha)$ (spherical harmonic)

- Whittaker function $W_{-\eta_\alpha, l+1/2}(2 q_\alpha r_\alpha) \to \exp(-q_\alpha r_\alpha)$
  with Sommerfeld parameter η_α, bound-state parameter q_α
  e.g. $\eta_{bx} = \frac{Z_b Z_x e^2 \mu_{bx}}{\hbar^2 q_{bx}} \qquad q_{bx} = \sqrt{2 \mu_{bx} S_{bx}}/\hbar$
  and separation energy S_{bx} of particle a into b and x
- asymptotic normalization coefficient (ANC) $C_\alpha^a(l)$

Wave Functions - Scattering States

general form of asymptotics (without particle spins)

$$\Psi_{bx}^{(+)} \to \frac{4\pi}{k_{bx}} \sum_{\alpha=(bx),(cy),\ldots} \phi_\alpha \frac{1}{r_\alpha} \sqrt{\frac{v_{bx}}{v_\alpha}} \sum_{lm} g_{\alpha l}^{(+)}(r_\alpha) i^l Y_{lm}(\hat{r}_\alpha) Y_{lm}^*(\hat{k}_{bx})$$ for $r_\alpha \to \infty$

with radial wave functions $$g_{\alpha l}^{(+)}(r_\alpha) = \frac{1}{2i}\left[S_{\alpha(bx)}^{l} u_l^{(+)}(\eta_\alpha, k_\alpha r_\alpha) - \delta_{\alpha(bx)} u_l^{(-)}(\eta_\alpha, k_\alpha r_\alpha)\right]$$

and angular parts $Y_{lm}(\hat{r}_\alpha)$, $Y_{lm}(\hat{k}_\alpha)$ (spherical harmonics)

- Coulomb wave functions
  $$u_l^{(\pm)}(\eta_\alpha, k_\alpha r_\alpha) = e^{\mp i\sigma_l}\left[G_l \pm iF_l\right] \to \exp\left\{\pm i\left[k_\alpha r_\alpha - 2\eta_\alpha \ln(k_\alpha r_\alpha) - \frac{l\pi}{2}\right]\right\}$$
  with Sommerfeld parameter η_α, momentum $\hbar\vec{k}_\alpha$, energy E_{bx} of relative motion
  e.g. $$\eta_{bx} = \frac{Z_b Z_x e^2 \mu_{bx}}{\hbar^2 k_{bx}} \qquad k_{bx} = \sqrt{2\mu_{bx}E_{bx}}/\hbar \qquad \mu_{bx} = \frac{m_b m_x}{m_b + m_x}$$
- S-matrix elements $S_{\alpha(bx)}^{l}$
  e.g. elastic scattering $S_{\alpha\alpha}^{l} = e^{2i[\sigma_l + \delta_l(\alpha)]}$ with Coulomb and nuclear phase shifts

Wave Functions - Cross Sections I

- extraction of scattering amplitude $f_{\alpha(bx)}$ for reaction $b + x \to c + y$

$$\Psi_{bx}^{(+)} - e^{i\vec{k}_{bx}\cdot\vec{r}_{bx}} \to \sum_{\alpha=(bx),(cy),\ldots} f_{\alpha(bx)} \frac{\exp(ik_\alpha r_\alpha)}{r_\alpha} \phi_\alpha$$ for $r_\alpha \to \infty$

$$\Rightarrow \quad f_{\alpha(bx)} = \frac{\sqrt{\pi}}{ik_{bx}} \sqrt{\frac{v_{bx}}{v_\alpha}} \sum_l \sqrt{2l+1} \left[S^l_{\alpha(bx)} - \delta_{\alpha(bx)} \right] Y_{l0}(\hat{r}_\alpha)$$ with $\vec{k}_{bx} = k_{bx}\vec{e}_z$

- differential reaction cross section

$$\frac{d\sigma}{d\Omega_\alpha}(b + x \to \alpha) = \frac{v_\alpha}{v_{bx}} \left| f_{\alpha(bx)} \right|^2 = \frac{\pi}{k_{bx}^2} \left| \sum_l \left[S^l_{\alpha(bx)} - \delta_{\alpha(bx)} \right] Y_{l0}(\hat{r}_\alpha) \right|^2$$

- total reaction cross section

$$\sigma(b + x \to \alpha) = \frac{\pi}{k_{bx}^2} \sum_l \left| S^l_{\alpha(bx)} - \delta_{\alpha(bx)} \right|^2$$

Wave Functions - Cross Sections II

- separation of Coulomb and nuclear contribution to S-matrix element
  $S^l_{\alpha\beta} = e^{i\sigma_l(\alpha)} S^{Nl}_{\alpha\beta} e^{i\sigma_l(\beta)}$ with Coulomb phase shifts $\sigma_l(\alpha)$, $\sigma_l(\beta)$

- decomposition of elastic scattering amplitude $(\alpha = \beta)$ $f_{\alpha\beta} = f^C_{\alpha\beta} + f^N_{\alpha\beta}$ with

  ◦ $$f^C_{\alpha\beta} = \frac{\sqrt{\pi}}{ik_\beta} \sum_l \sqrt{2l+1} \left\{ \exp\left[2i\sigma_l(\beta)\right] - 1 \right\} Y_{l0}(\hat{r}_\beta)$$
  $$= -\frac{\eta_\beta}{2k_\beta} \left(\sin \frac{\vartheta_\beta}{2} \right)^{-2} \exp\left(2i\sigma_0(\beta) - 2i\eta_\beta \ln \sin \frac{\vartheta_\beta}{2} \right)$$

  $\Rightarrow$ Rutherford cross section $\frac{d\sigma_R}{d\Omega_\alpha} = \left| f^C_{\alpha\beta} \right|^2 = \frac{\eta^2_\beta}{4k^2_\beta} \left(\sin \frac{\vartheta_\beta}{2} \right)^{-4}$

  ◦ $$f_{\alpha\beta} = \frac{\sqrt{\pi}}{ik_\beta} \sum_l \sqrt{2l+1} \exp\left[2i\sigma_l(\beta)\right] \left[S^{Nl}_{\alpha\beta} - 1 \right] Y_{l0}(\hat{r}_\alpha)$$

  $\Rightarrow$ fast convergence

Wave Functions - Penetrability

effect of Coulomb barrier
(and centrifugal barrier)
$\Rightarrow$ reduced probability of finding
the particles at small distance R

- compare modulus squared of scattering wave functions at $r = R$ and $r \to \infty$
  $\Rightarrow$ penetrability factor

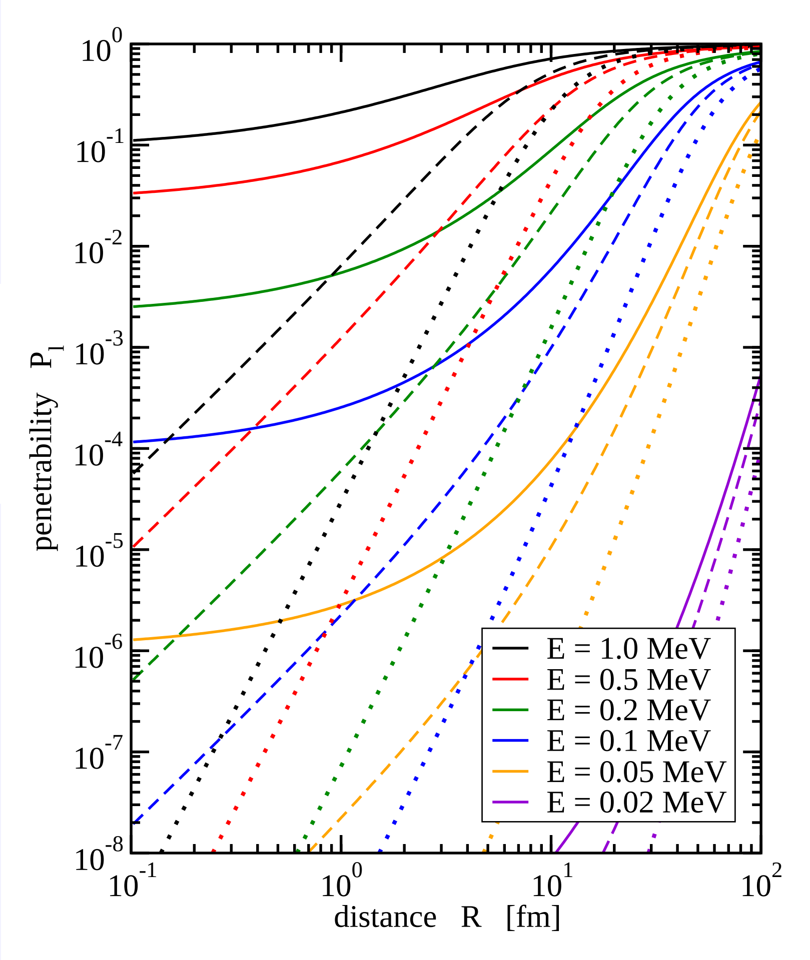
$$P_l(R) = \frac{\lim_{r\to\infty} \left| u_l^{(\pm)}(\eta; kr) \right|^2}{\left| u_l^{(\pm)}(\eta; kR) \right|^2} = \frac{1}{F_l^2(\eta, kR) + G_l^2(\eta, kR)}$$

- s-wave scattering ($l = 0$):

$$\lim_{R\to 0} P_0(R) = \frac{2\pi\eta}{\exp(2\pi\eta) - 1}$$

$\Rightarrow$ cf. definition of S factor

example: p + ^{7}Be scattering
solid lines: $l = 0$
dashed lines: $l = 1$
dotted lines: $l = 2$



Wave Functions - Radial Integrals I

reduced electric transition matrix element

$$\langle k l_f || \mathcal{M}(E\lambda) || l_i \rangle \propto \frac{4\pi}{k} Z_{\text{eff}}^{(\lambda)} e I_{l_i}^{l_f}(\lambda)$$ with $$I_{l_i}^{l_f}(\lambda) = \int dr \, h_{l_i}^{l_f}(\lambda)$$

asymptotic form of integrand in radial integral $I_{l_i}^{l_f}(\lambda)$

$$h_{l_i}^{l_f}(\lambda) \to g_{l_f}^{(-)*}(r) \, r^{\lambda} \, f_{l_i}(r)$$

depends only on

- Whittaker function in bound state (q, η_i, l_i)
- Coulomb wave functions in scattering state $(k, \eta_f = \frac{q}{k}\eta_i, l_f)$
- asymptotic normalization factor (ANC) $C_i(l_i)$ of bound state
- phase shift δ_{l_f} of scattering state

weakly bound nuclei

- radial integral $I_{l_i}^{l_f}(\lambda)$ dominated by asymptotic contributions
- effect of final-state interaction small $\Rightarrow$ phase shifts small
- absolute scaling determined by ANC

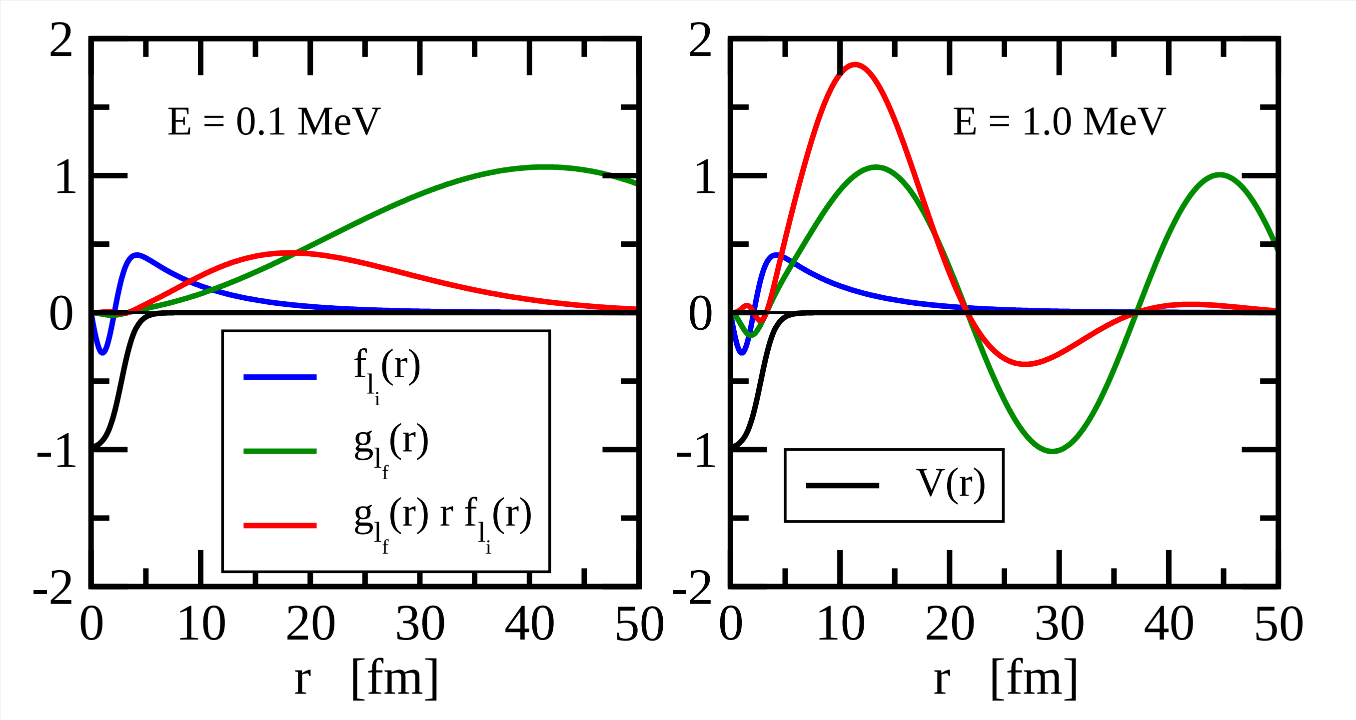
Wave Functions - Radial Integrals II

- **example**: breakup of ^{11}Be $\rightarrow$ ^{10}Be $+ n$

  neutron halo nucleus with neutron separation energy $S_n = 0.504$ MeV

  $E1$ transition from s wave bound state to p wave scattering state with energy E

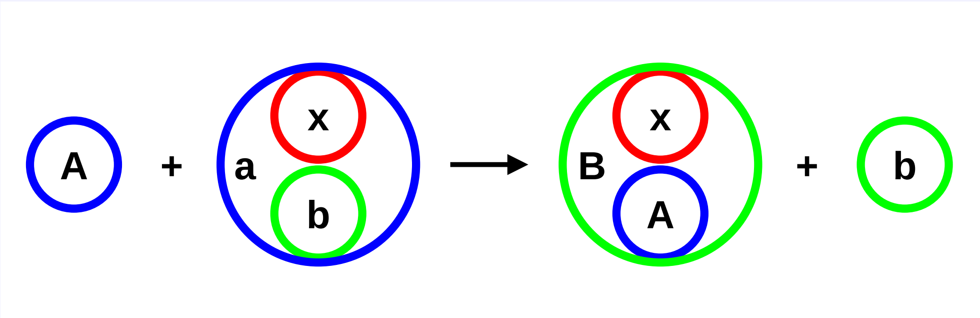
  $\Rightarrow$ integrand in radial integral

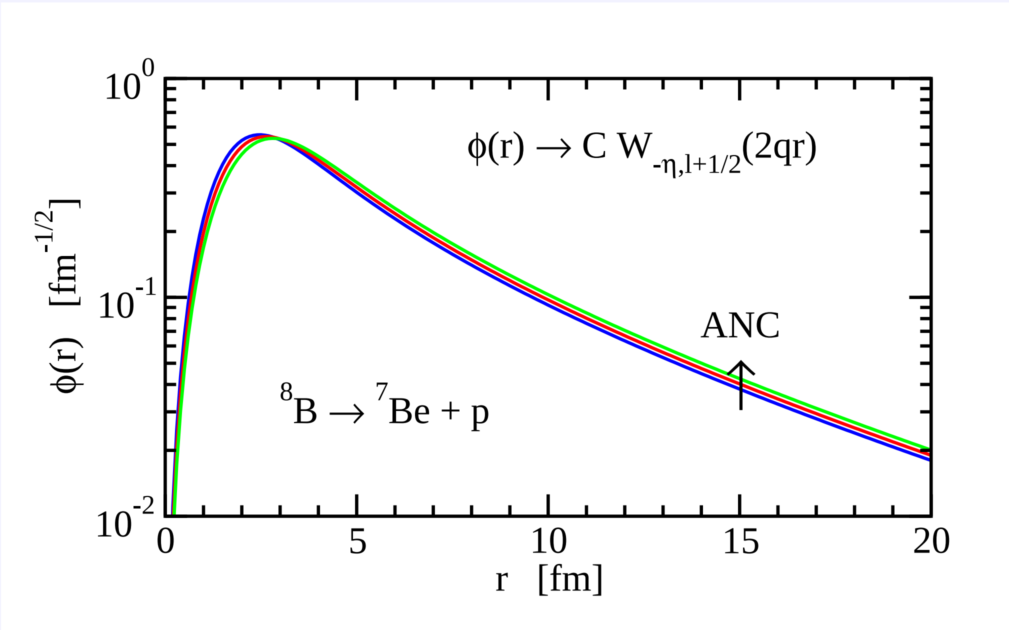


- Eλ transitions at low relative energies

  $\Rightarrow$ matrix elements determined by asymptotic of wave functions

ANC Method - Idea





extract asymptotic normalization coefficient (**ANC**)

for breakup of nucleus B into $A + x$
or nucleus a into $b + x$

from cross section of transfer reaction

$$A + a \rightarrow B + b$$

with $a = b + x$ and $B = A + x$

$$\Downarrow$$

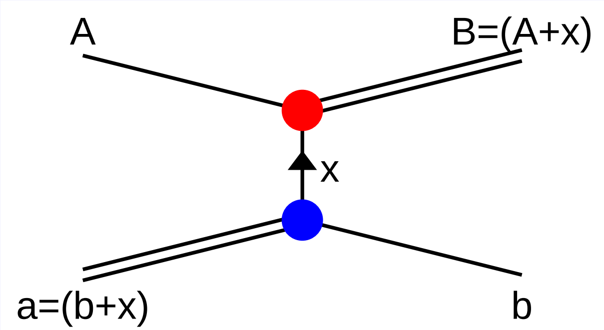
calculate astrophysical S factor $S(E)$

in the limit $E \rightarrow 0$

(H.M. Xu et al., Phys. Rev. Lett. 73 (1994) 2027)

ANC Method - Theory I

- T-matrix element in post-form DWBA
- replace exact overlap functions by asymptotic form with asymptotic normalization coefficients (ANCs) and Whittaker functions



- **overlap functions** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$$\langle \phi_b | \phi_a \rangle \approx \frac{C^a_{bx}(l_a)}{r_{bx}} W_{-\eta_{bx}, l_a+1/2}(2 q_{bx} r_{bx}) Y_{l_a m_a}(\hat{r}_{bx}) \phi_x$$

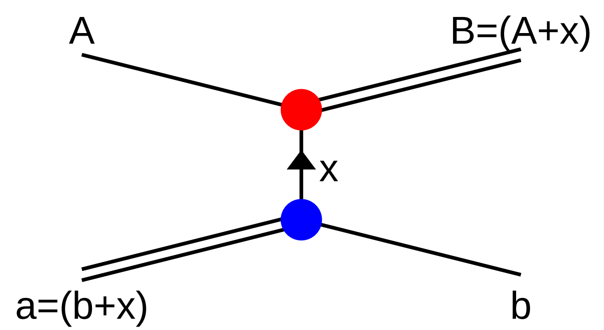
$$\langle \phi_A | \phi_B \rangle \approx \frac{C^B_{Ax}(l_B)}{r_{Ax}} W_{-\eta_{Ax}, l_B+1/2}(2 q_{Ax} r_{Ax}) Y_{l_B m_B}(\hat{r}_{Ax}) \phi_x$$

- **cross section** of transfer reaction to bound state

$$\frac{d\sigma}{d\Omega_{Bb}} = \left| C^a_{bx} \right|^2 \left| C^B_{Ax} \right|^2 \frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$ with reduced DWBA cross section $$\frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$

- two ANCs appear corresponding to two poles in diagram
- ANC/Whittaker functions replace spectroscopic factors/full single-particle wave functions in conventional DWBA

ANC Method - Theory II



- approximations only valid for weakly bound states/ peripheral reactions
- precise optical potentials for $A + a$ and $B + b$ scattering required
- one additional ANC needed

$\Rightarrow$ calculate low-energy S factor of capture reaction $b(x, \gamma)a$ numerically

with extracted ANC $C^a_{bx}(l_a)$ and asymptotic wave function

- $|C^a_{bx}|^2 \Leftrightarrow S(0)$ unique relation?
- effect of final-state interaction V_{bx}?

  $\Rightarrow$ systematic model calculations

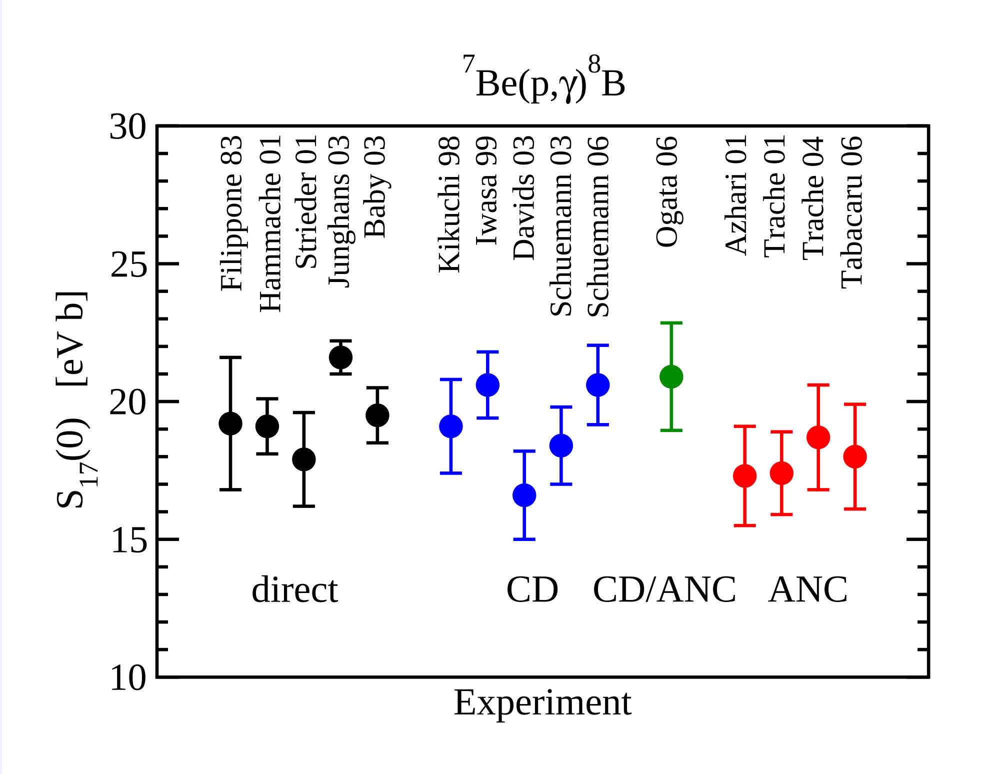
ANC Method - Example: ^{7}Be(p,γ)^{8}B

experiments: (Texas A&M University)

extraction of ANC from

- proton transfer reactions
  ^{10}B(^{7}Be,^{8}B)^{9}Be, ^{14}N(^{7}Be,^{8}B)^{13}C
  with 85 MeV ^{7}Be beam
  A. Azhari et al., PRC 63 (2001) 055803
  G. Tabacaru et al., PRC 73 (2006) 025808
- breakup ^{8}B $\rightarrow$ ^{7}Be+p
  on C, Si, Sn, and Pb targets with
  beam energies from 30 to 1000 A MeV
  L. Trache et al., PRL 87 (2001) 271102,
  PRC 69 (2004) 032802
- neutron transfer reaction
  ^{13}C(^{7}Li,^{8}Li)^{12}C with 63 MeV ^{7}Li beam
  and charge symmetry
  L. Trache et al., PRC 67 (2003) 062801(R)

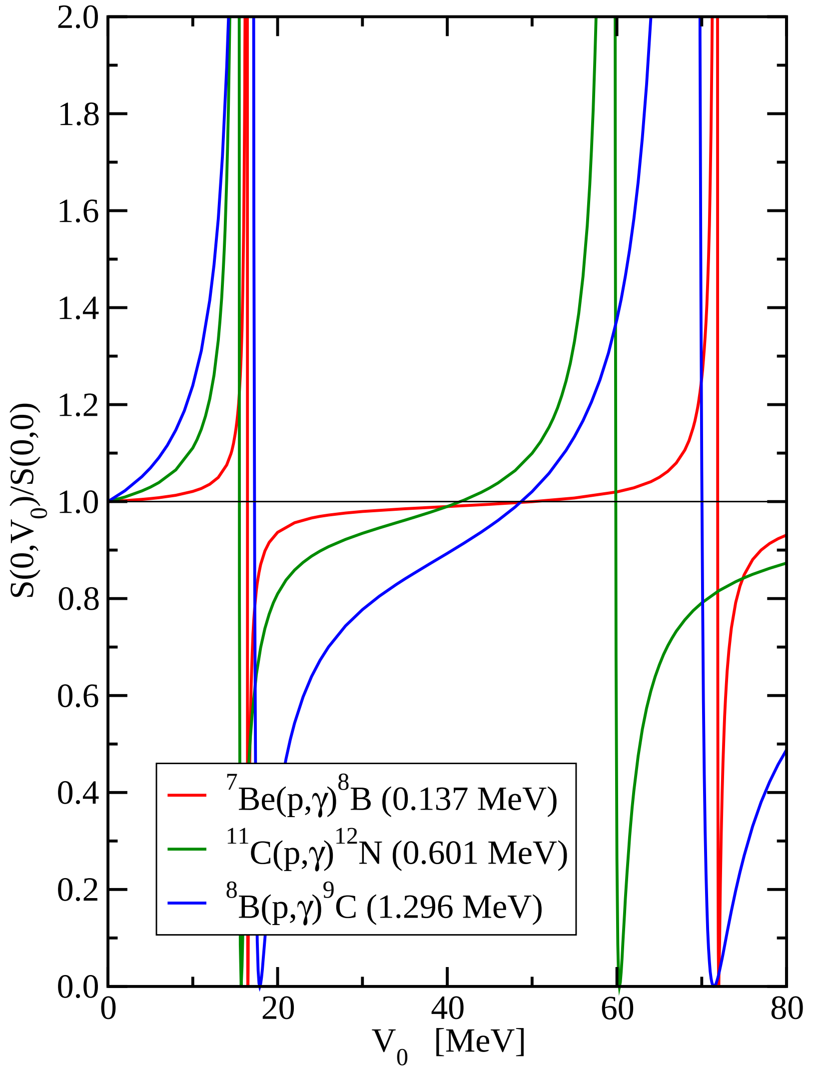
comparison with other methods



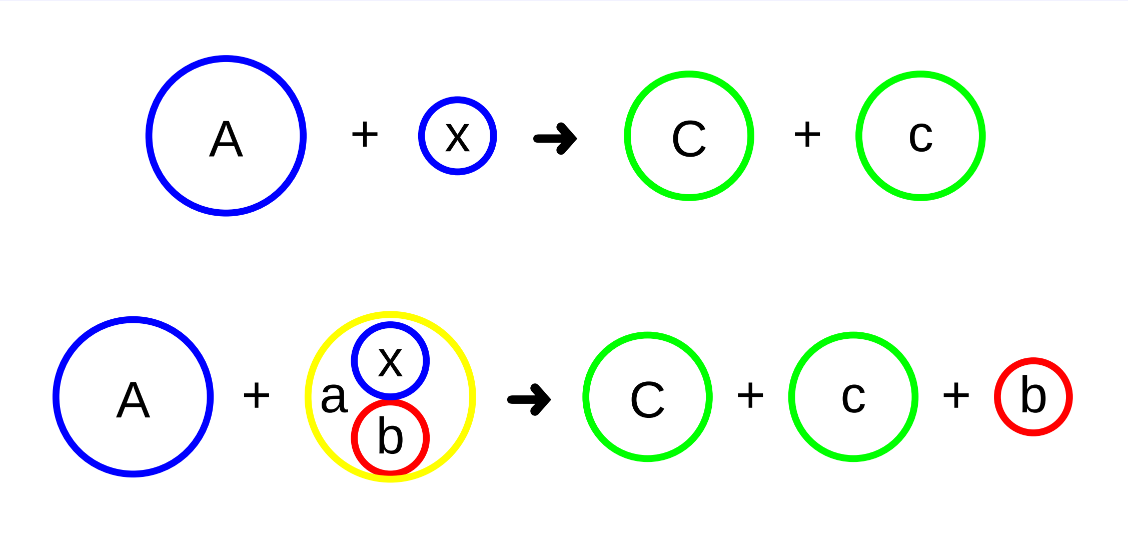
ANC Method - Continuum Interaction

- effects of interaction in continuum states
  - modification of shape of cross section, S factor (i.e. energy dependence)
  - change of $S(0)$ even though $\delta \to 0$
- calculation of zero-energy S factor $S(0)$ in single-particle model with Woods-Saxon potential with different depths V_0
- example: $E1\ s \to p$ wave capture for different nuclei with proton+core structure
  $\Rightarrow$ stronger variation of $S(0)$ with V_0 with larger proton separation energy
- simple relation ANC $\Leftrightarrow S(0)$ only correct for weakly bound nuclei

S. Typel and G. Baur, Nucl. Phys. A 759 (2005) 245



Trojan-Horse Method - Idea



replace **two-body reaction**

$$A + x \rightarrow C + c$$

by **three-body reaction**

$$A + a \rightarrow C + c + b$$

with **Trojan horse** $a = b + x$

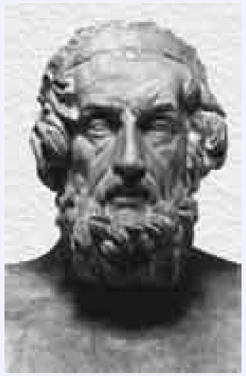
and **spectator** b

- small momentum transfer to spectator
  $\Rightarrow$ quasi-free scattering dominates
- large relative energy of system $A + a$
  $\Rightarrow$ no suppression of cross section
  $\Rightarrow$ no electron screening
- small relative energies of system $A + x$ accessible
  $\Rightarrow$ application to nuclear astrophysics

(G. Baur, Phys. Lett. B 178 (1986) 35)

$\ldots\ \kappa\epsilon\kappa\alpha\lambda\upsilon\mu\mu\epsilon\nu o\iota\ \iota\pi\pi o.$

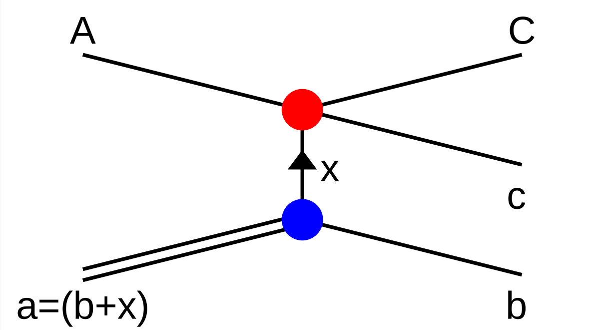
Homer, Odyssey VIII, 503



Trojan-Horse Method - Theory I

- T-matrix element in post-form DWBA with $B = C + c$

$$T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb} - U_{Bb} | \phi_a \phi_A \chi_{Aa}^{(+)} \rangle$$



- use asymptotic form of scattering wave function $\phi_B = \Psi_{Cc}^{(-)}$ in reaction channel $C + c \to A + x$ (essential "surface approximation")

$\Rightarrow$ **overlap function** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$$\langle \phi_A | \Psi_{Cc}^{(-)} \rangle \approx \frac{4\pi}{k_{Cc} r_{Ax}} \sqrt{\frac{v_{Cc}}{v_{Ax}}} \sum_{lm} \xi_l^*(r_{Ax}) i^l Y_{lm}(\hat{r}_{Ax}) Y_{lm}^*(\hat{k}_{Cc}) \phi_x$$

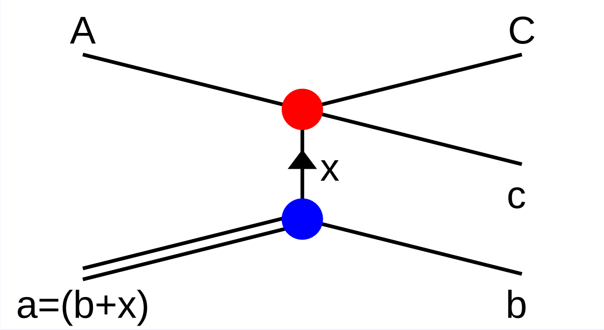
with $\xi_l(r_{Ax}) = \frac{1}{2i} \left[S_{AxCc}^l u_l^{(+)}(\eta_{Ax}; k_{Ax} r_{Ax}) - \delta_{AxCc} u_l^{(-)}(\eta_{Ax}; k_{Ax} r_{Ax}) \right]$

and S-matrix element S_{AxCc}^l of reaction $C(c, x)A$

general theory: S. Typel and G. Baur, Ann. Phys. (N.Y.) 305 (2003) 228

Trojan-Horse Method - Theory II

- post-form DWBA T-matrix element with surface approximation for $ax \neq Cc$
  $\Rightarrow$ factorization



$$T_{(Bb)(Aa)} \propto \sum_{lm} S^{l*}_{AxCc}$$

$$\times \langle \frac{u_l^{(+)}(\eta_{Ax};k_{Ax}r_{Ax})}{k_{Cc}r_{Ax}} Y_{lm}(\hat{r}_{Ax})\phi_x\phi_b\chi^{(-)}_{Bb}|V_{Bb} - U_{Bb}|\phi_a\chi^{(+)}_{Aa}\rangle$$

$\Rightarrow$ **cross section** of transfer reaction to continuum (single channel, $Ax \neq Cc$)

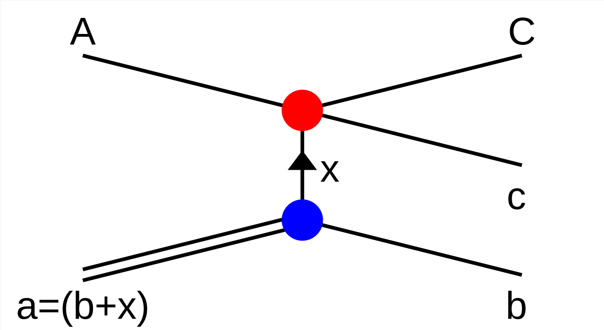
$$\frac{d^3\sigma}{d\Omega_{Bb}d\Omega_{Cc}dE_{Cc}} = \left|S^l_{AxCc}\right|^2 \frac{d^3\tilde{\sigma}_l}{d\Omega_{Bb}d\Omega_{Cc}dE_{Cc}}$$

with reduced DWBA cross section

- S-matrix elements S^l_{AxCc} determine cross section

$$\frac{d\sigma}{d\Omega_{Ax}}(C + c \to A + x) = \frac{\pi}{k^2_{bx}} \left|\sum_l S^l_{AxCc} Y_{l0}(\hat{r}_{Ax})\right|^2$$

Trojan-Horse Method - Theory III



additional approximations
(not necessary in general, but convenient)

- potential $V_{Bb} - U_{Bb} = V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi^{(-)}_{Bb}$, $\chi^{(+)}_{Aa}$

- T-matrix element in PWBA with surface approximation

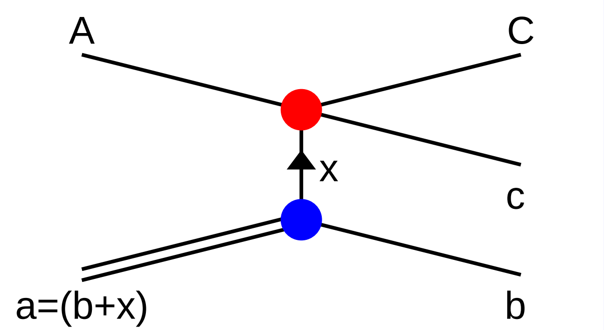
$$T_{(Bb)(Aa)} \propto \sum_{lm} S^{l*}_{AxCc} \langle \phi_x \phi_b \exp\left(i\vec{Q}_{Bb} \cdot \vec{r}_{bx}\right) |V_{xb}| \phi_a \rangle$$
$$\times \langle \frac{u^{(+)}_l(\eta_{Ax}; k_{Ax} r_{Ax})}{k_{Cc} r_{Ax}} Y_{lm}(\hat{r}_{Ax}) | \exp\left(i\vec{Q}_{Aa} \cdot \vec{r}_{Ax}\right) \rangle$$

with momentum transfers

$$\vec{Q}_{Aa} = \vec{k}_{Aa} - \frac{\mu_{Ax}}{m_x} \vec{k}_{Bb} \qquad \vec{Q}_{Bb} = \vec{k}_{Bb} - \frac{\mu_{bx}}{m_x} \vec{k}_{Aa}$$

- factorization with three factors:
  - two factors for two poles in diagram
  - additional matrix element, depends on kinematics (η_{Ax}!)

Trojan-Horse Method - Theory IV



additional approximations
(not necessary in general, but convenient)

- potential $V_{Bb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi^{(-)}_{Bb}$, $\chi^{(+)}_{Aa}$

$\Rightarrow$ **cross section** of transfer reaction to continuum (single channel)

$$\frac{d^3\sigma}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}} = K\, W(\vec{Q}_{Bb}) \frac{d\sigma_l}{d\Omega}(Ax \to Cc)\, T_l(k_{Ax})$$

with kinematic factor K

- **momentum distribution** $W(\vec{Q}_{Bb}) = |\tilde{\Phi}^a_{bx}(\vec{Q}_{Bb})|^2$

  depending on momentum transfer to spectator $b \Rightarrow$ quasi-free scattering conditions

- **cross section** $\frac{d\sigma_l}{d\Omega}(Ax \to Cc)$ of two-body reaction

- **penetration factor** $T_l(k_{Ax}) \approx k^3_{Ax} \exp(2\pi\eta_{Ax})$

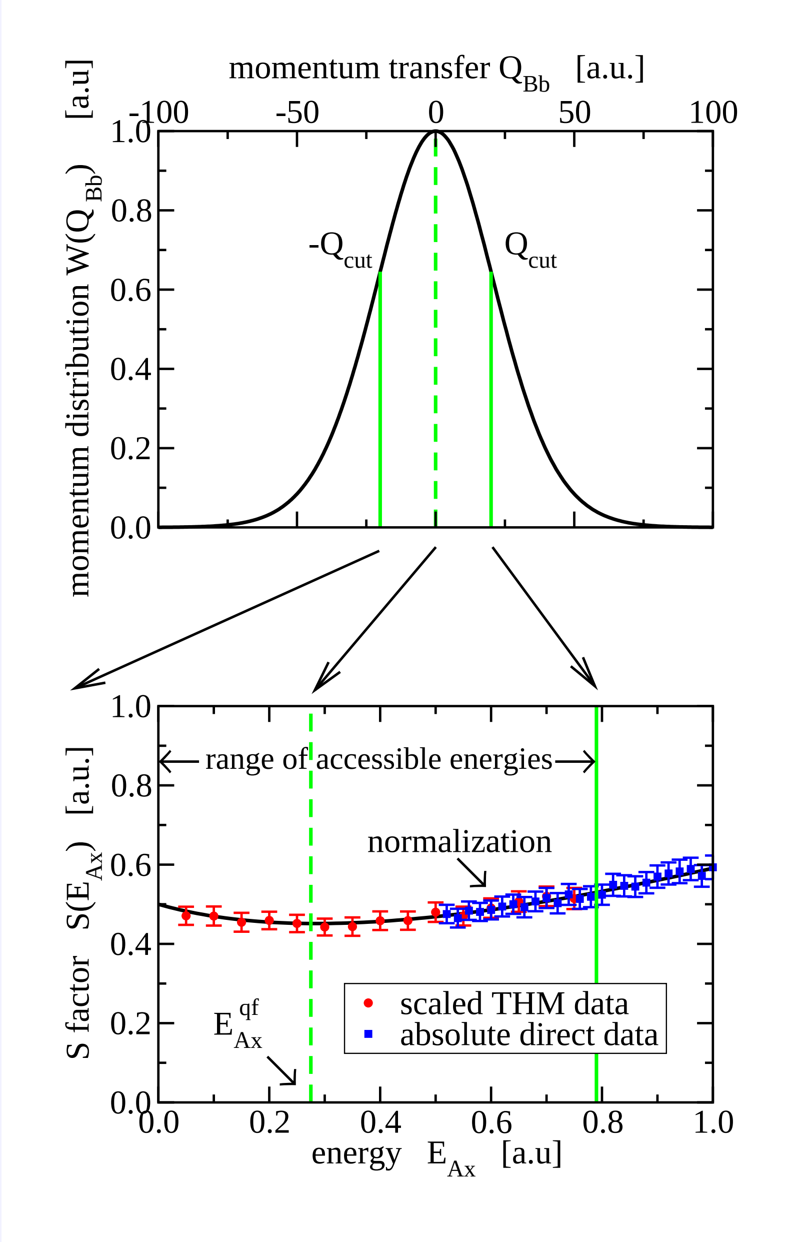
$\Rightarrow$ cancels suppression of two-body cross section by Coulomb barrier for $E_{Ax} \to 0$

Trojan-Horse Method - Application

- selection of Trojan horse $a = b + x$
  (e.g. ^{2}H $= n + p$, ^{6}Li $= \alpha + d$, ...)
  with binding energy $\epsilon_a > 0$ and
  well known ground state wave function
  $\Rightarrow$ momentum distribution $W(\vec{Q}_{Bb})$
- width of momentum distribution W
  $\Leftrightarrow$ Fermi motion of x inside a
- condition $\vec{Q}_{Bb} = 0$ defines
  "quasi-free energy" in $A + x$ system

$$E_{Ax}^{qf} = E_{Aa}\left(1 - \frac{\mu_{Aa}}{\mu_{Bb}}\frac{\mu_{bx}^2}{m_x^2}\right) - \epsilon_a \ll E_{Aa}$$

- cutoff in $\vec{Q}_{Bb}$ determines range of
  accessible energies E_{Ax} around E_{Ax}^{qf}
- small momentum transfer
  $\Rightarrow$ dominance of quasi-free process
- normalization of cross section to direct data
  at higher E_{Ax}



Trojan-Horse Method - Example: ^{2}H(^{6}Li,α)^{4}He

- **direct reaction**: ^{2}H(^{6}Li,α)^{4}He
  - experiment with gas target

    (S. Engstler et al., Z. Phys. A 342 (1992) 471)
  - $S(0) = 17.4$ MeV b

    (corrected for electron screening)

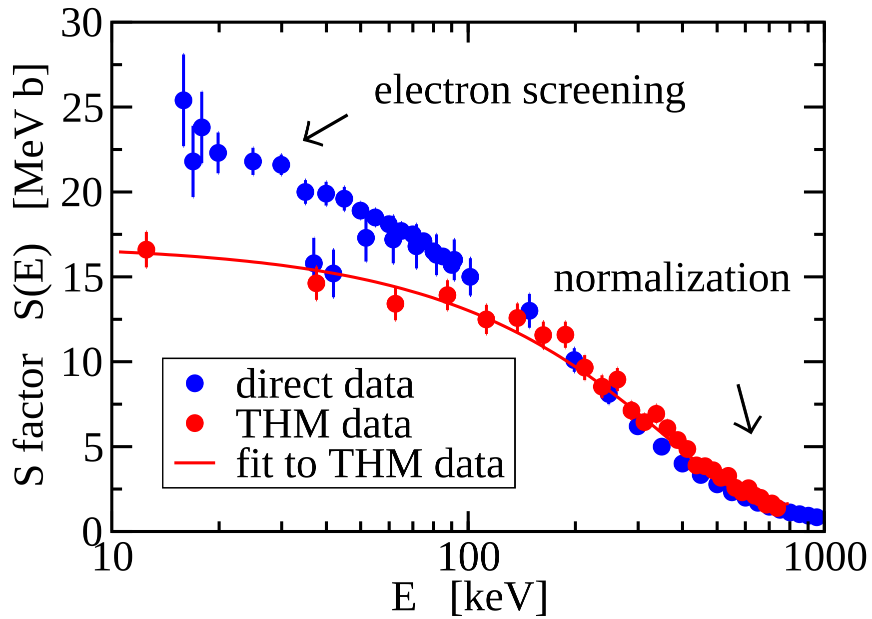
- **THM**: ^{6}Li(^{6}Li,$\alpha\alpha$)^{4}He
  - experiment with 6 MeV ^{6}Li beam

    (C. Spitaleri et al., Phys. Rev. C 63 (2001) 055801;

    A. Musumarra et al., Phys. Rev. C 64 (2001) 068801)
  - $E^{qf} = 25$ keV
  - target and projectile breakup
  - $l = 0$, $\hbar Q_{Bb} < 35$ MeV/c
  - normalization to direct data

    for $E > 600$ keV

    $\Rightarrow S(0) = (16.9 \pm 0.5)$ MeV b



- **electron screening potential**:

  U_e(direct) $= (330 \pm 120)$ eV

  U_e(THM) $= (320 \pm 50)$ eV

  U_e(theory) $= 186$ eV (adiabatic limit)

Trojan-Horse Method - Example: ^{6}Li(p,α)^{3}He

- **direct reaction**: ^{6}Li(p,α)^{3}He
  - experimental data

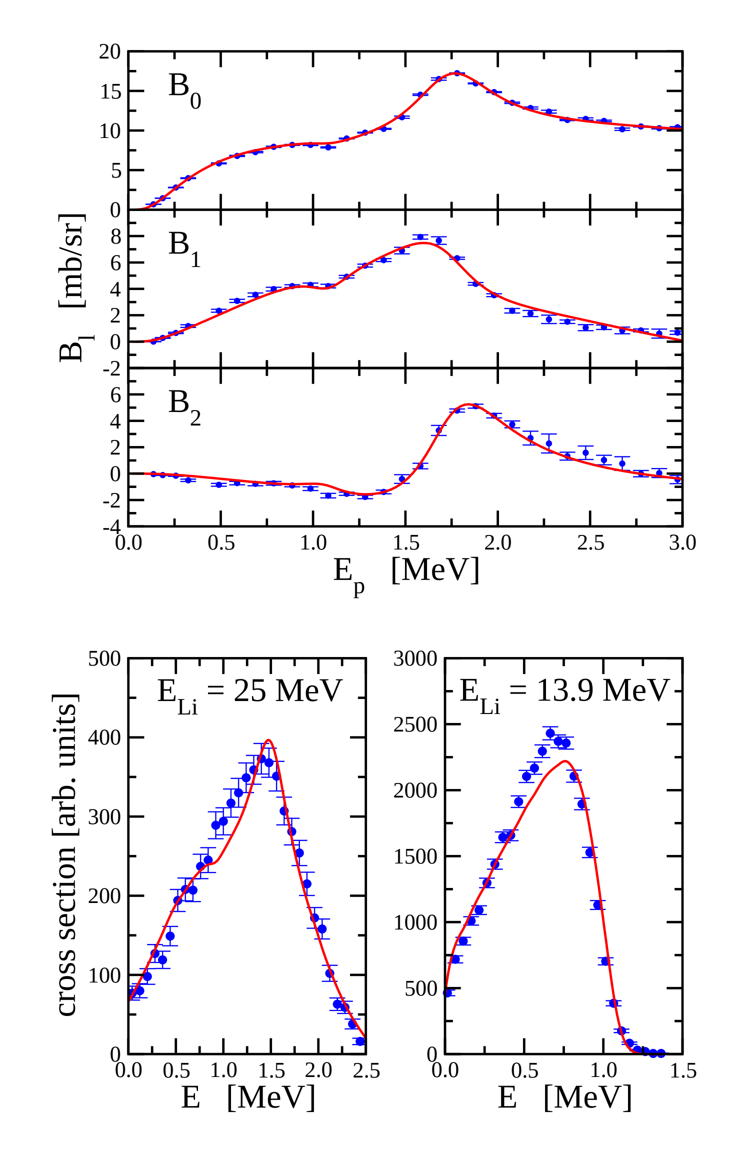
    (J. Elwyn et al., Phys. Rev. C 20 (1979) 1084)
  - differential cross section

    $$d\sigma/d\Omega = \sum_l B_l P_l(\cos\theta)$$
  - non-resonant s wave and resonant p wave contribution
  - S matrix from R-matrix fit

    $\Rightarrow$ simulation of THM experiment

- **THM**: ^{2}H(^{6}Li,α^3He)n
  - experiments with 13.9/25 MeV ^{6}Li beam

    (A. Tumino et al., Phys. Rev. C 67 (2003) 065803 and preliminary results)
  - $E^{qf} = -0.24/1.35$ MeV
  - $\hbar Q_{Bb} < 30$ MeV/c
  - finite cross section at $E = 0$ MeV!



Trojan-Horse Method

- analysis only in simple theoretical approximations
  $\Rightarrow$ full DWBA calculations needed for quasi-free scattering conditions
  with consistent treatment of bound/scattering/resonant states
  (numerically very demanding)

- finite cross section at $E_{Ax} = 0 \Rightarrow$ continue to $E_{Ax} < 0$:
  investigation of subthreshold resonances

- extension to radiative capture reactions possible
  $\Rightarrow$ additional approach independent from Coulomb dissociation and ANC methods

- study elastic scattering without Coulomb contribution $\Rightarrow$ optical potentials

- application to reactions with exotic nuclei $\Rightarrow$ large cross sections

- extracted S factor not affected by electron screening
  $\Rightarrow$ determination of electron screening potential U_e by comparison to direct data
  $\Rightarrow$ consistent values for U_e, larger than adiabatic limit, challenge for theory

Conclusions

- **Indirect methods** provide complementary information on reactions of astrophysical interest
  - Coulomb dissociation method
  - method of asymptotic normalization coefficients (ANC)
  - Trojan-Horse method
- similar characteristics and theoretical concepts
- importance of nuclear reaction theory
  - direct reactions with certain kinematical conditions
  - peripheral reactions, asymptotics of wave functions
  - approximations $\Rightarrow$ range of validity, accuracy
- great potential for future applications (nuclear astrophysics, structure and reactions of exotic nuclei, . . .)
- dedicated theoretical investigations in close collaboration with experiment