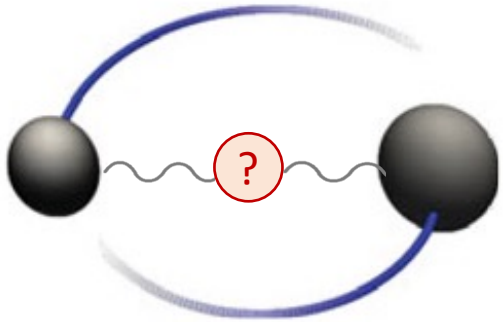
Causality bounds on gravitational EFTs

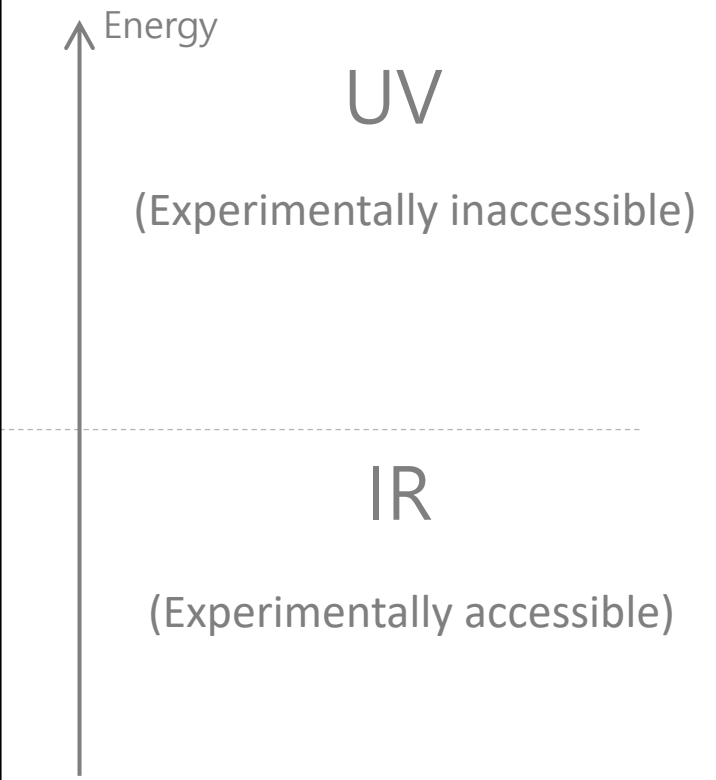
Scott Melville

Queen Mary University of London



EFT and Multi-Loop Methods for Advancing Precision in Collider and Gravitational Wave Physics (Oct 2024)

The EFT mindset: *learn about new high-energy physics by measuring EFT coefficients*



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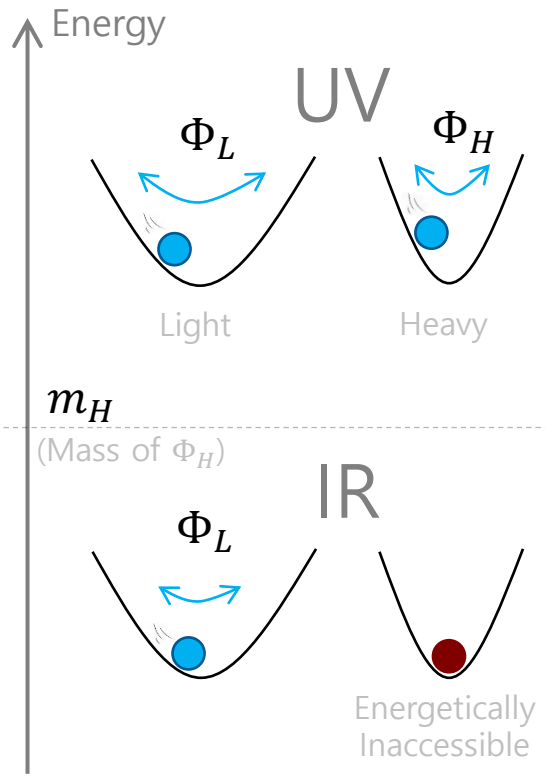


The EFT mindset: *learn about new high-energy physics by measuring EFT coefficients*

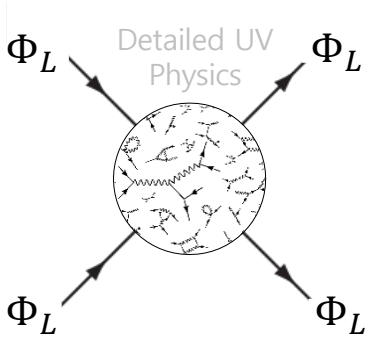
Energy

Φ_L Φ_H

$\mathcal{L}_{UV}[\Phi_L, \Phi_H]$

All degrees of freedom

Harmonic Oscillators



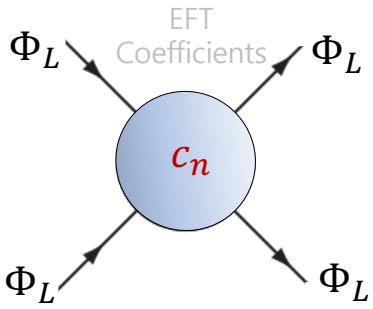
$$\mathcal{L}_{\text{UV}} = g_3 \Phi_L^2 \Phi_H + g_4 \Phi_L^2 \Phi_H^2 + \cdots$$

Φ_L

$\mathcal{L}_{EFT}[\Phi_L]$

Only light degrees of freedom

Energetically Inaccessible



$$\mathcal{L}_{\text{EFT}} = c_1 \Phi_L^4 + c_2 \partial^2 \Phi_L^4 + \cdots$$

small small small

The EFT mindset: *learn about new high-energy physics by measuring EFT coefficients*

Energy

$\mathcal{L}_{UV}[\Phi_L, \Phi_H]$

All degrees of freedom

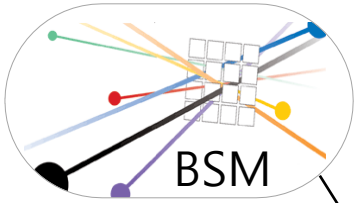
Harmonic Oscillators

$\mathcal{L}_{EFT}[\Phi_L]$

Only light degrees of freedom

Energetically Inaccessible

Two important applications



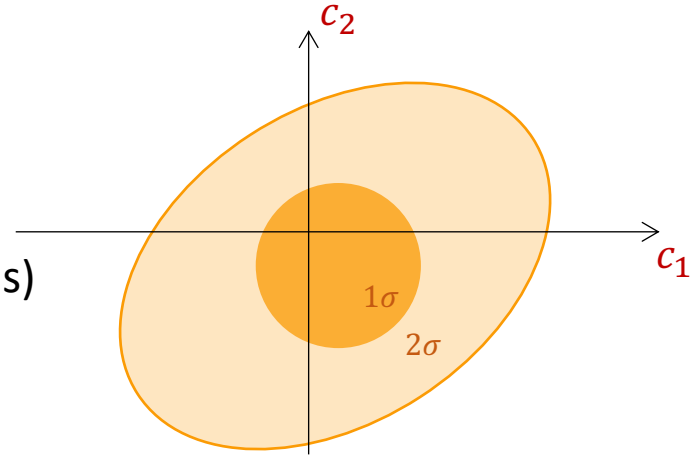
$$\mathcal{L}_{\text{GREFT}} = \mathcal{L}_{\text{GR}}[g_{\mu\nu}] + \sum_n c_n \mathcal{O}_n[g_{\mu\nu}]$$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}[\psi] + \sum_n c_n \mathcal{O}_n[\psi]$$

The EFT mindset: $\mathcal{L}_{\text{EFT}} = \sum_n c_n \mathcal{O}_n$ provides model-agnostic template for data analysis

However, two common problems:

(i) Large experimental uncertainty
(can be difficult to measure even first EFT corrections)



(ii) Suppose we measure $c_2 = 1/(100\ \text{TeV})$. What does this tell us about the UV physics?

Rules out many UV completions…

New heavy states below/around 100 TeV

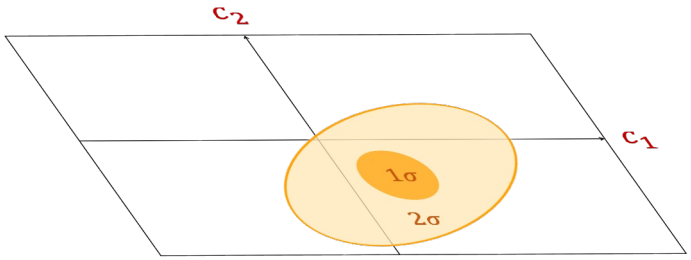
~~First heavy state much above 100 TeV~~

… but what else can it tell us about the heavy states?

The central argument for today's talk:

High energy

Low energy



$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm GR} + \underbrace{c_1\delta\mathcal{L}_1 + c_2\delta\mathcal{L}_2 + \cdots}_{\text{small corrections}}$$

(fixed by low-energy degrees of freedom + symmetries)

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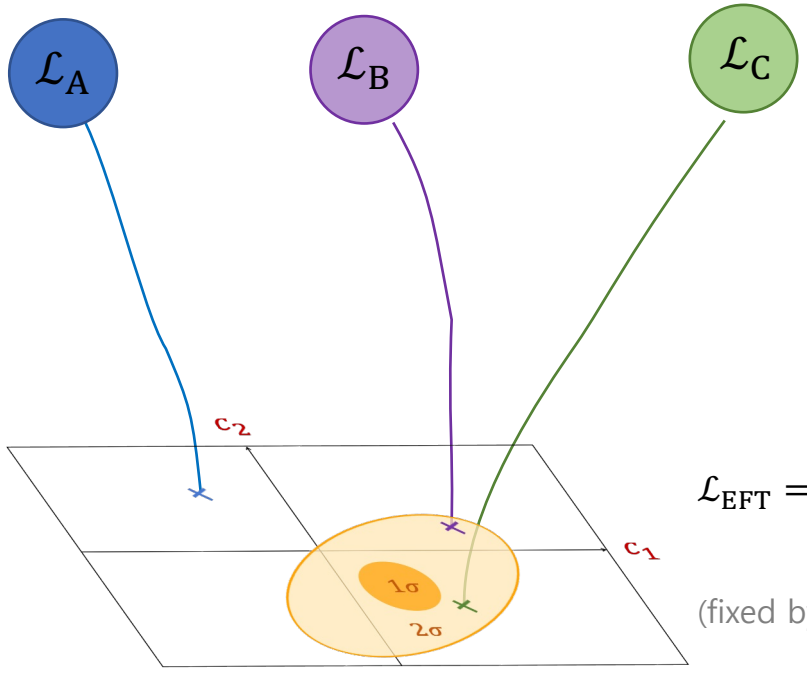

Particular UV theories correspond to points in the EFT parameter space

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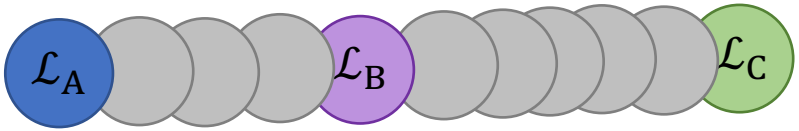

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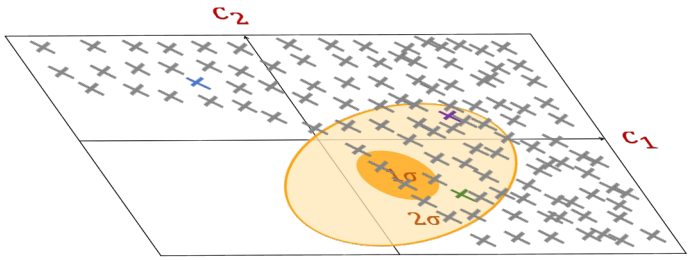

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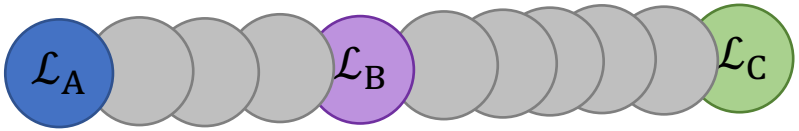

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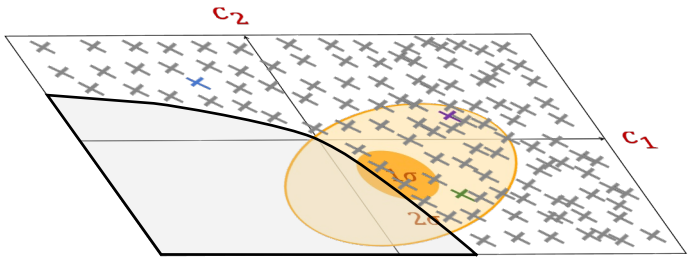

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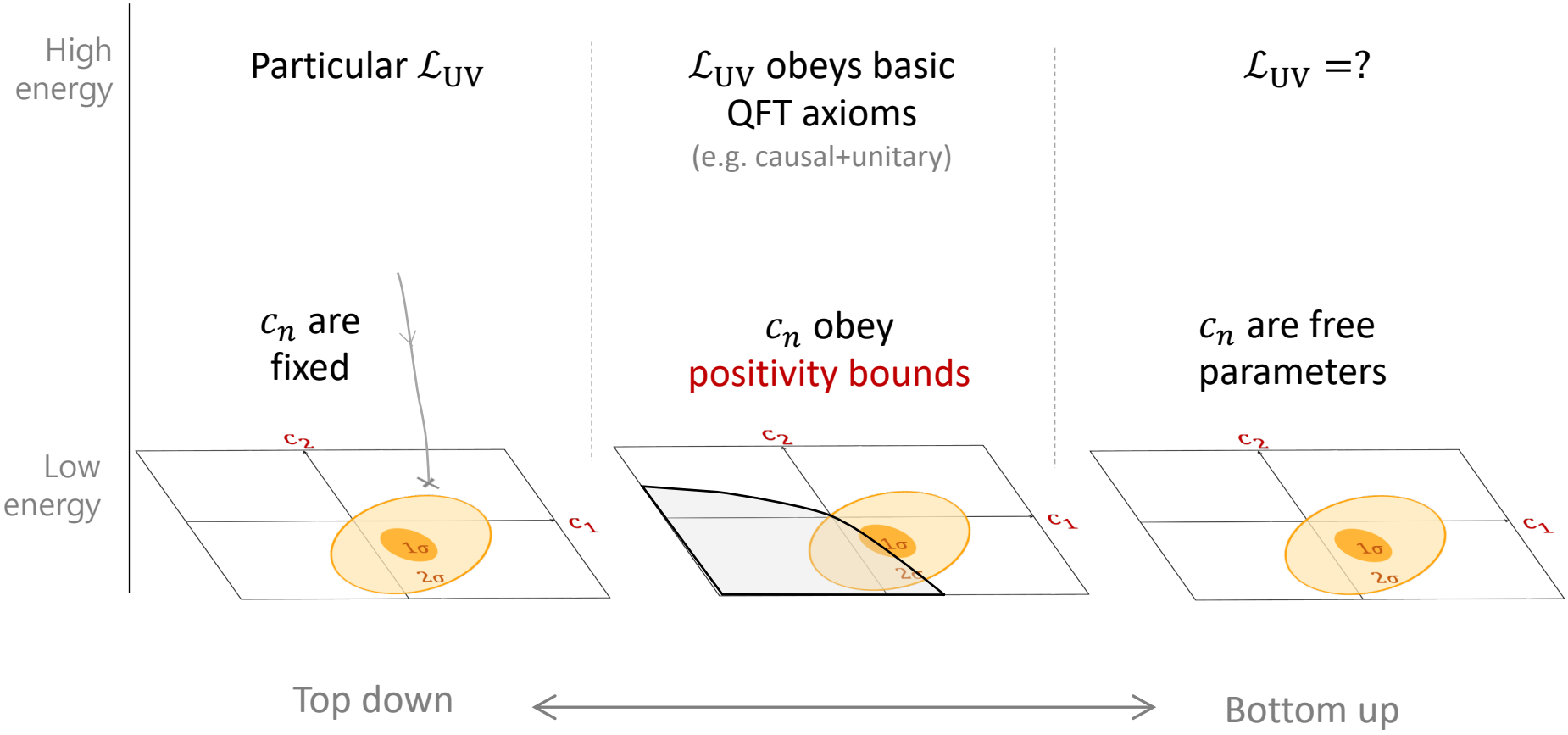
(fixed by low-energy degrees of freedom + symmetries)

Some EFT parameter values have no UV completion

The central argument for today's talk:

Positivity bounds use basic QFT axioms to rule out EFTs with no UV completion

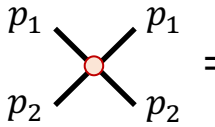


The central argument for today's talk:

If the UV completion is:

- unitary (positive probabilities)
- causal (analytic amplitudes)
- local (Froissart bounded)

then EFT amplitudes obey "positivity bounds":

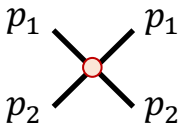
e.g. 

$= c_1 + c_2 s^2 + \mathcal{O}(s^4)$

total energy
$s = (p_1 + p_2)^2$

has $c_2 > 0$

If EFT coefficients violate positivity

e.g. 

has $c_2 \leq 0$

then there is no UV completion which is:

- unitary
- causal
- local

[Pham, ..., 1960's], ..., [Adams++, 2006], [Nicolis++, 2009], ...
[Arkani-Hamed, Bellazzini, Caron-Huot, de Rham, Huang, Melville, Rattazzi, Tolley, Zhou, ..., 2017-2022]

Outline

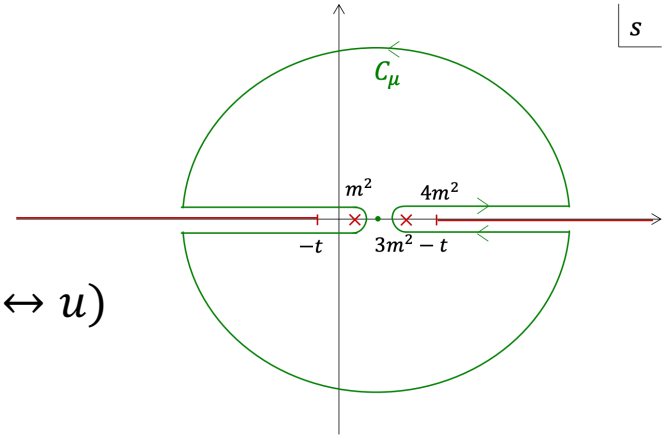
1) EFT positivity (without gravity)

2) EFT positivity for gravitational waves

UV properties ⇒ *Properties of* $2 \to 2$ *amplitude*

1. Lorentz symmetry $A(s,t)$
2. Causality
3. Locality

$$\partial_s^2 A(s_0,0) = \int_{4m^2}^{\infty} \frac{ds}{2\pi i} \frac{\text{Im } A(s,0)}{(s-s_0)^3} + (s \leftrightarrow u)$$



4. Unitarity ⇒ $\text{Im } A(s,0) > 0$

Therefore, $\partial_s^2 A(0,0) > 0$

$A(s,t) = c_0 + c_2 s^2 + \cdots$ has EFT coefficient $c_2 > 0$

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 2006]

1. Lorentz symmetry
2. Causality (Analyticity)
3. Locality (Bounded)

$$D[A] = \oint F[A]$$

Argument can be extended in many different ways

4. Unitarity $\Rightarrow$ $F[A] > 0$

Therefore, $D[A] > 0$ (i.e. bounds on *every* EFT coef $c_{n\geq 2}$)

[Adams et al, 2006] Lower bounds on $\partial_s^{2n} A|_{t=0}$

…

[SM, de Rham, Tolley, Zhou 2017] Lower bounds on $\partial_t^{2m}\partial_s^{2n} A|_{t=0}$

…

[Bellazzini, Elias, Miro, Rattazzi, Riembau, Riva 2011.00037] Two-sided bounds

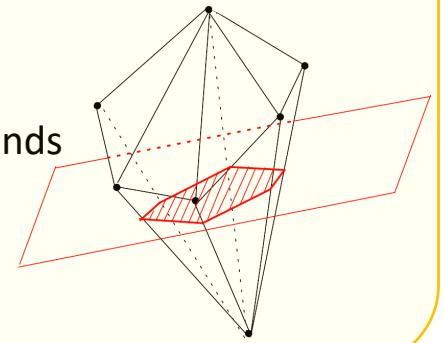
[Tolley, Wang, Zhou 2011.02400] Two-sided bounds

[Caron-Huot, van Duong 2011.02957] Two-sided SDP bounds

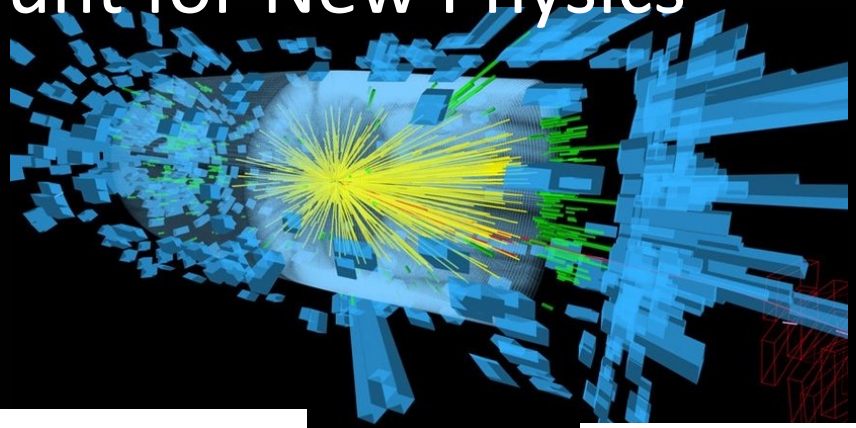
[Arkani-Hamed, Tzu-Chen Huang, Yu-tin Huang 2012.15849] "EFThedron"

…

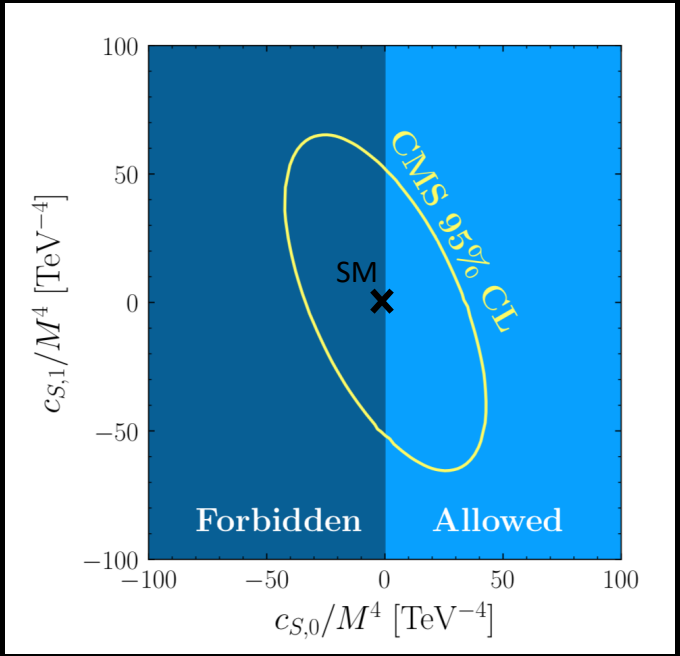
[de Rham, Reece, Tolley, Zhou 2022] Snowmass review



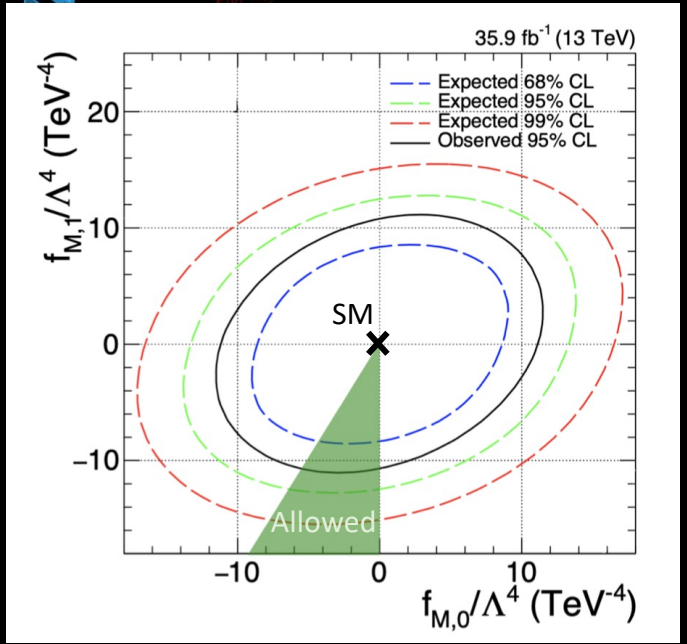
EFTs and the Hunt for New Physics



[aQGC, Remmen+Rodd 2020]



[VBS, Zhang+Zhou 2018]



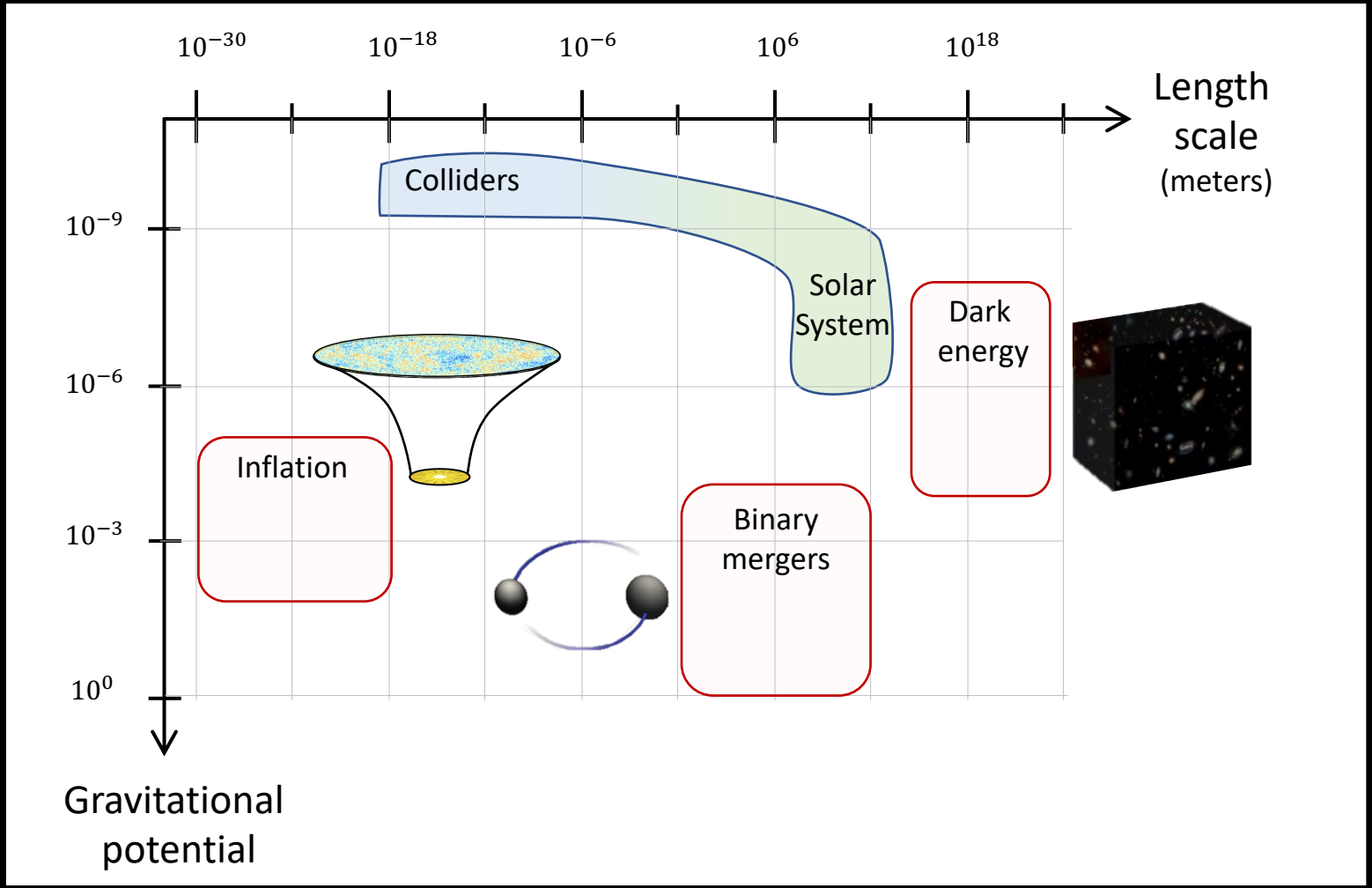
EFTs and the Hunt for New Physics

CMB

LSS

GWs

EFTs and the Hunt for New Physics



EFT Positivity for GWs

GR

EFT corrections

$$S_{\text{GREFT}} = \int d^4x \sqrt{-g}\, \frac{M_P^2}{2} \left[R + \frac{b_1}{\Lambda^4} R^{(3)} + \frac{c_1}{\Lambda^6} \left(R^{(2)} \right)^2 + \frac{c_2}{\Lambda^6} \left(\tilde{R}^{(2)} \right)^2 \right]$$

$$R^{(2)} = R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu} \,, \qquad \tilde{R}^{(2)} = \frac{1}{2} R^{\mu\nu}{}_{\alpha\beta} \epsilon^{\alpha\beta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu} \,, \qquad R^{(3)} = R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu}$$

GR

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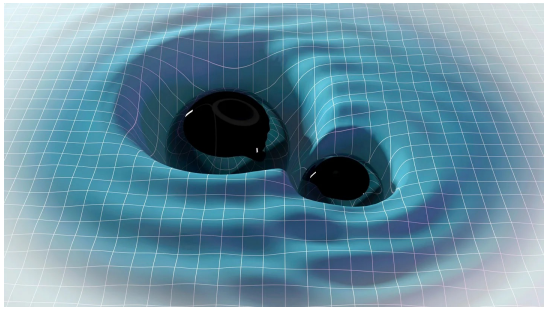

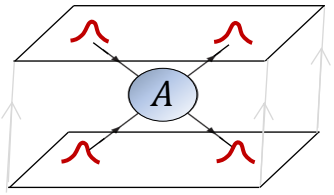

≈ Minkowski

GR

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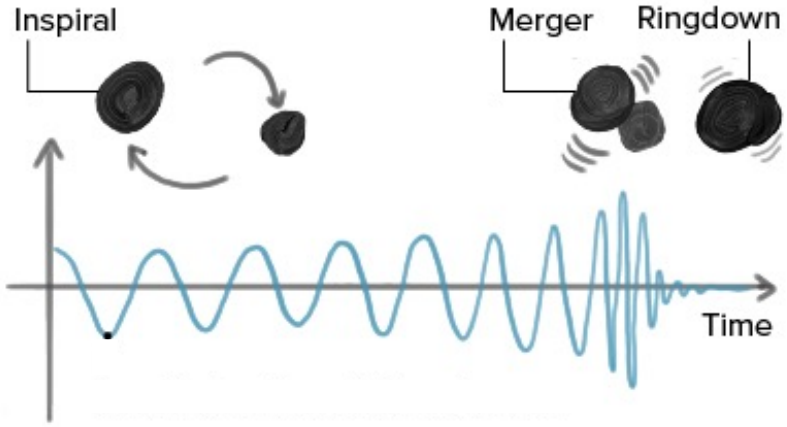

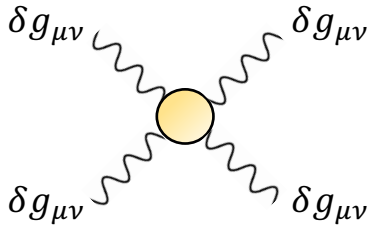

Use observed waveform to constrain EFT coefficients

Use unitarity/causality/locality to constrain EFT coefficients

GR

EFT corrections

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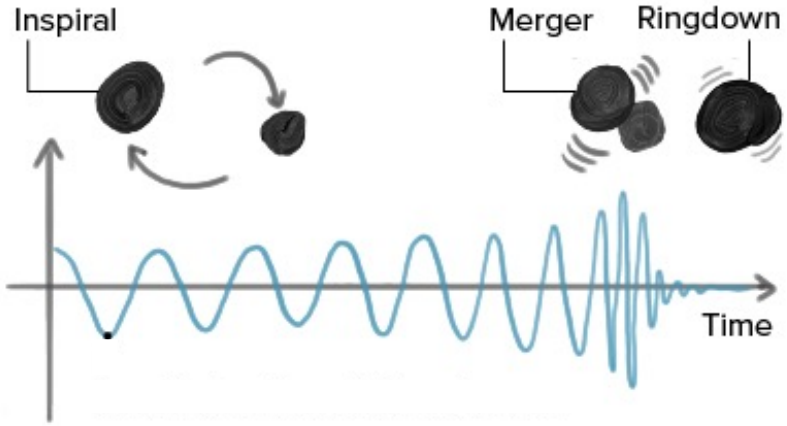
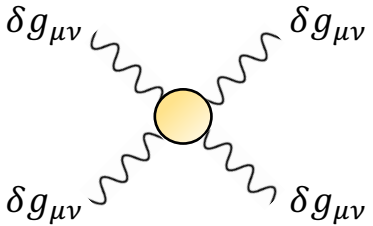




Use observed waveform to constrain EFT coefficients

learn about UV

inform data analysis

Use unitarity/causality/locality to constrain EFT coefficients

$\epsilon_s \sim 1/GM\Lambda$

To estimate size of corrections:

$$\frac{\frac{1}{\Lambda^4}\sqrt{-g}R^3}{\sqrt{-g}R} \sim \frac{\frac{1}{\Lambda^4}(\partial^2 h)^3}{h\partial^2 h} \sim \frac{1}{(\Lambda r)^4}\frac{GM}{r} \sim \epsilon_s^4\left(\frac{v^2}{c^2}\right)^5$$

$$\frac{\frac{1}{\Lambda^6}\sqrt{-g}R^4}{\sqrt{-g}R} \sim \frac{\frac{1}{\Lambda^6}(\partial^2 h)^4}{h\partial^2 h} \sim \frac{1}{(\Lambda r)^6}\left(\frac{GM}{r}\right)^2 \sim \epsilon_s^6\left(\frac{v^2}{c^2}\right)^8$$

If EFT resolves BH horizon, then corrections are heavily suppressed

$1 \lesssim R_S\,\Lambda \sim \frac{1}{\epsilon_s} \quad \Rightarrow \quad R^3$ is 5PN* and R^4 is 8PN effect

If EFT only resolves waveform, then corrections can be observed

$(GM\pi f)^{2/3} \lesssim \frac{1}{\epsilon_s} \quad \Rightarrow \quad R^3$ is 1PN* and R^4 is 2PN effect

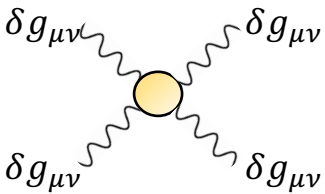
*Actually 1 order higher due to cancellations

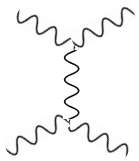
$$S_{\text{GREFT}} = \int d^4x \sqrt{-g}\, \frac{M_P^2}{2} \left[R + \frac{b_1}{\Lambda^4} R^{(3)} + \frac{c_1}{\Lambda^6} \left(R^{(2)} \right)^2 + \frac{c_2}{\Lambda^6} \left(\tilde{R}^{(2)} \right)^2 \right]$$

$S_{\text{UV}} =$ Unitary, causal, local (i.e. ∃ smooth, bounded extension of A with pos disc)

EFT coefficients *should* be constrained by UV properties…

… but positivity with gravity is complicated by ``t channel pole''



$\supset$  $\propto \frac{s^2}{M_P^2\, t} \Rightarrow \partial_s^2 A|_{t=0}$ is not well-defined

$$S_{\text{GREFT}} = \int d^4x \sqrt{-g} \frac{M_P^2}{2} \left[R + \frac{b_1}{\Lambda^4} R^{(3)} + \frac{c_1}{\Lambda^6} \left(R^{(2)} \right)^2 + \frac{c_2}{\Lambda^6} \left(\tilde{R}^{(2)} \right)^2 \right]$$

Various strategies for tackling t channel pole:

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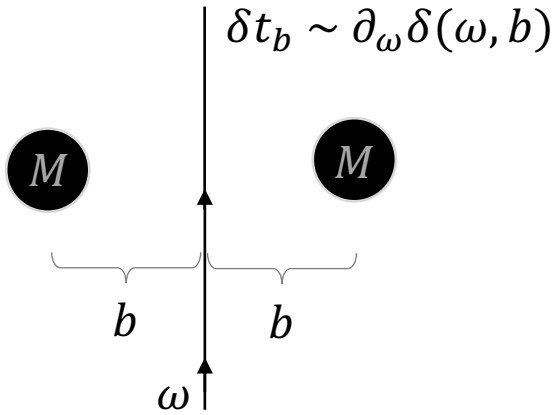
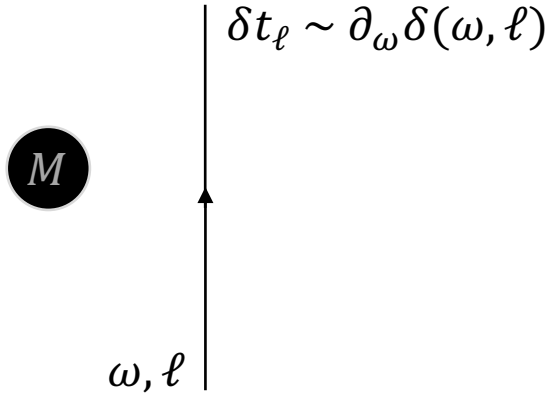


[Camanho, Maldacena, Edelstein, Zhiboedov 1407.5597]



[de Rham, Tolley 1909.00881]

Causality $\Rightarrow$ No resolvable time advance $\omega\, \delta t \lesssim 1$

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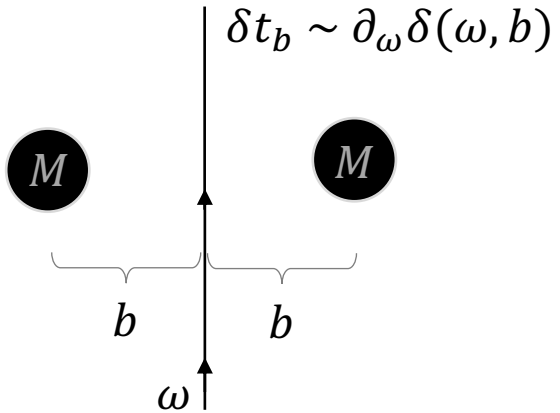

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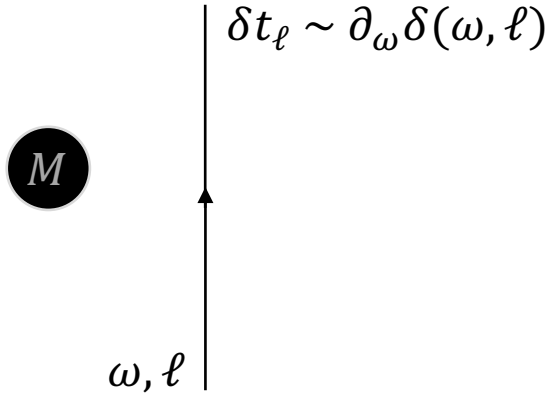

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Various strategies for tackling t channel pole:

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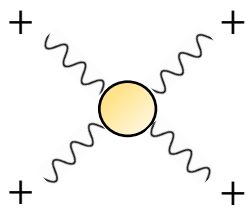
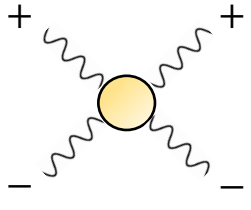
1) Find observable with no pole,

e.g. $\partial_s^4 A|_{t=0}$ is finite & positive

[Bellazzini, Cheung, Remmen, 2016]



$c_1 > 0$



$c_2 > 0$

2 polarisations $\Rightarrow$ 2 different bounds

$$S_{\text{GREFT}} = \int d^4x \sqrt{-g}\, \frac{M_P^2}{2} \left[R + \frac{b_1}{\Lambda^4} R^{(3)} + \frac{c_1}{\Lambda^6} \left(R^{(2)} \right)^2 + \frac{c_2}{\Lambda^6} \left(\tilde{R}^{(2)} \right)^2 \right]$$

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2) Use finite impact parameter (introduce IR cut-off m_{IR})

$$4(c_1 - c_2) \leq 12.3 \log\left(\frac{M}{m_{\text{IR}}}\right) - 13.5$$

[Caron-Huot, Li, Parra-Martinez, Simmons-Duffin, 2023]

$$S_{\text{GREFT}} = \int d^4x \sqrt{-g}\, \frac{M_P^2}{2} \left[R + \frac{b_1}{\Lambda^4} R^{(3)} + \frac{c_1}{\Lambda^6} \left(R^{(2)} \right)^2 + \frac{c_2}{\Lambda^6} \left(\tilde{R}^{(2)} \right)^2 \right]$$

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1) Find observable with no pole, e.g. $\partial_s^4 A|_{t=0}$ $\Rightarrow$ $c_1 > 0,\ c_2 > 0$

2) Use finite impact parameter (introduce IR cut-off m_{IR}) $\Rightarrow$ $c_1 - c_2 > \#$

3) Assume Regge growth at high energies

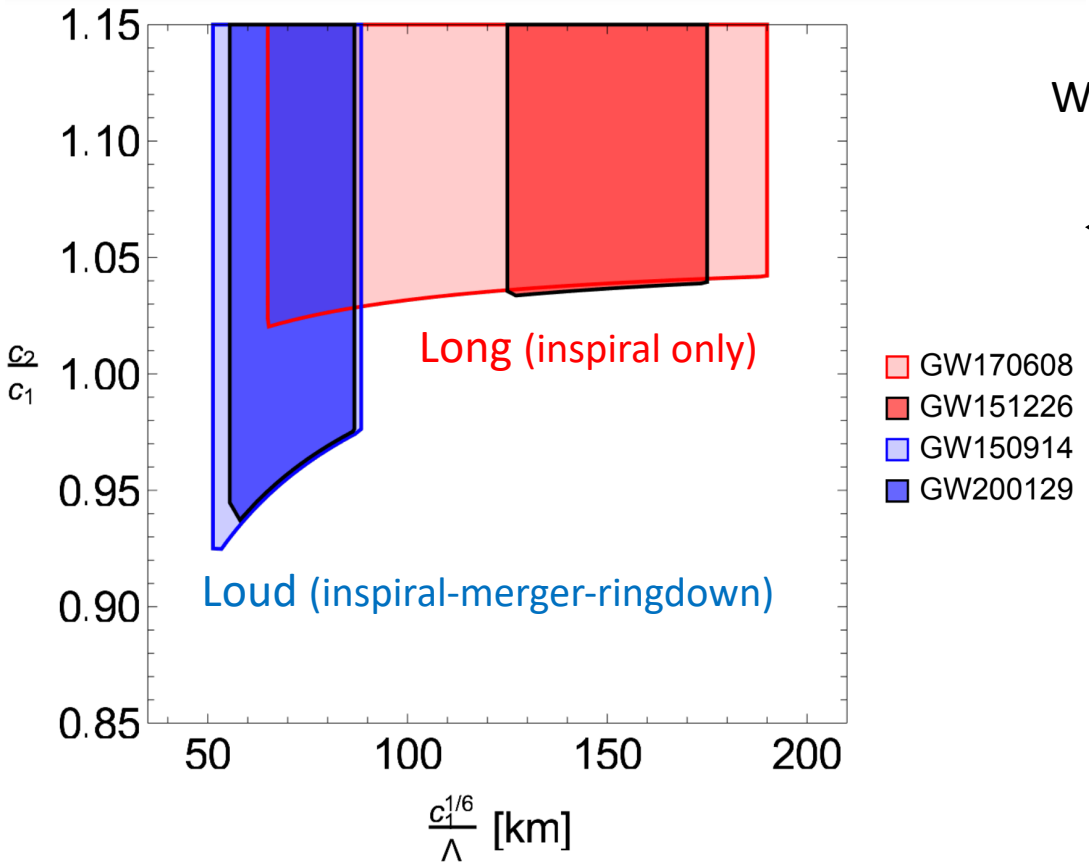
$$A(s,t) \sim s^{\alpha(t)} \quad \Rightarrow \quad \partial_s^2 [A(s,t) - \text{pole}]_{t=0} > -O\left(\frac{1}{\Lambda^2 M_P^2} \right)$$

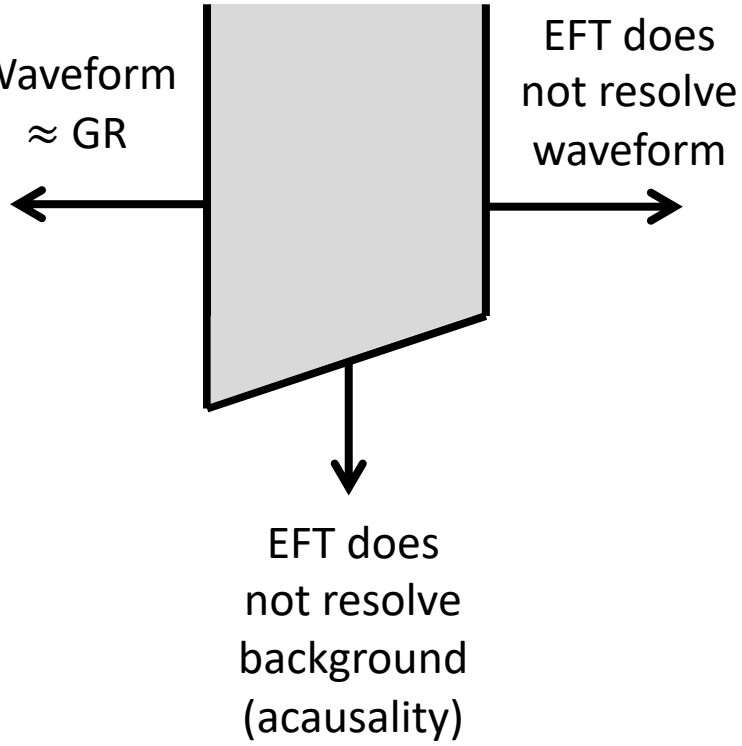
[Tokuda, Aoki, Hirano, 2020]

Gravity allows small violation of positivity

Remarkably: all bounds point in ≈ same direction [Melville 2401.05524]

Observational Constraints from Gravitational Waves





[Melville, 2401.05524]
[Silva, Ghosh, Buonanno 2205.05132]
[Sennett, Brito, Buonanno, Gorbenko, Senatore 1912.09917]

Comments

(i) Have neglected b_1 (since causality $\Rightarrow$ $b_1 < \mathcal{O}(c_1, c_2)$)

(ii) Stronger than solar system / laboratory bounds
(since local curvature scale $\sim 10^{-8}$/km vs $\sim 10^{-1}$/km near black hole)

(iii) Rules out theories with very low cut-off ($\sim 10^{-12}$ eV)

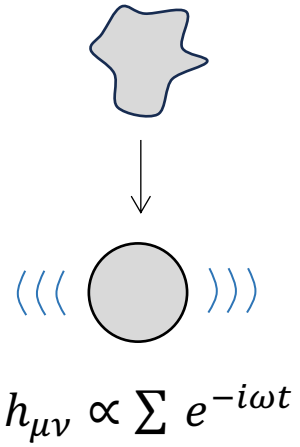
(iv) Assumes "soft UV completion", typical in cosmological EFTs

(v) Simplest UV completions ruled out by solar system tests

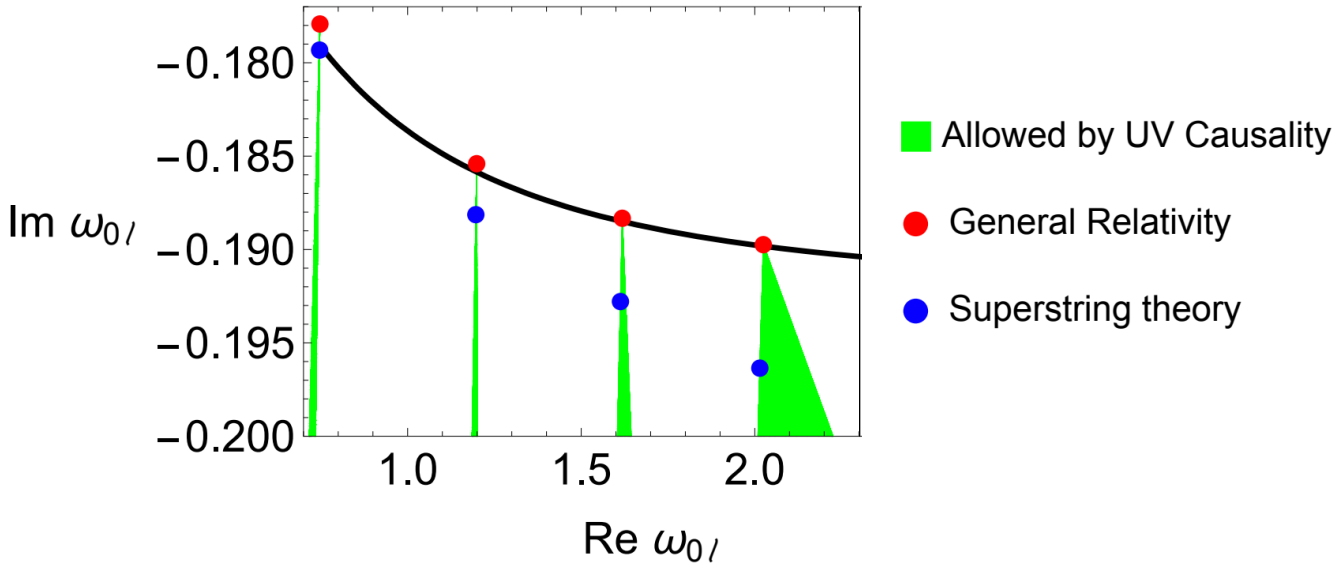
[Cassem, Hertzberg 2408.12118]

Comments

(vi) Causality/positivity bounds ⇒ BHs are *more stable* in the EFT [Melville, 2401.05524]



$h_{\mu\nu} \propto \sum e^{-i\omega t}$



(vii) For higher-derivative operators $C_q \left(R^{(2)}\right)^q$

Causality (BH stability) ⇒ $C_q > 0$

Summary

UV principles
(causality/unitarity/locality)

⇒

Bounds on
EFT coefficients

Space of consistent Effective Field Theories is

- mathematically interesting
- phenomenologically important