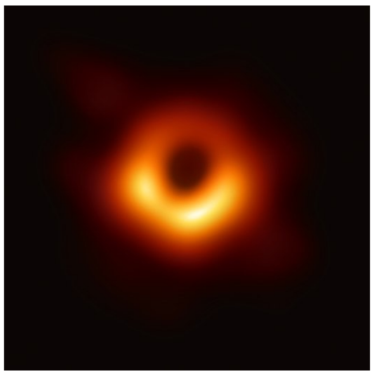
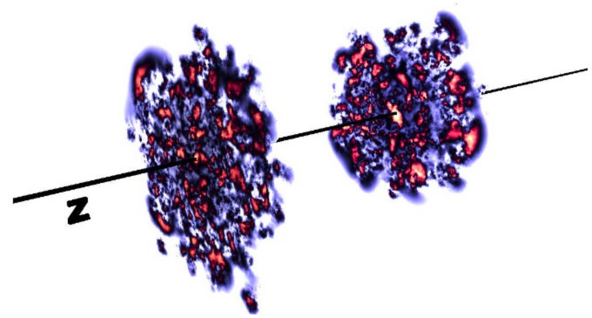
$2\rightarrow$ N Regge amplitudes in QCD and gravity: universal features and emergent dynamics



$M_{BH}=(6.5 \pm 0.2_{stat} \pm 0.7_{sys}) \times 10^9\, M_\odot$
at center of Messier 87
Event Horizon Telescope image of photon ring

10^9 km

10^{-19} km



Shockwave collisions of Color Glass Condensate gluon states in nuclei, arXiv:1206.6805

Raju Venugopalan

Brookhaven National Laboratory

CFNS, Stony Brook University

MIAPbP workshop, October 14-25, 2024

Based in part on recent published work (arXiv:2311.03463, 2312.03507, 2312. 11652, 2406.10483) with Himanshu Raj (Simons Confinement+ QCD Strings Collaboration Fellow at Stony Brook)

Talk outline

2 →N scattering in QCD in the Regge regime. RG evolution from low to high occupancy and emergent dynamics

From amplitudes to shockwave scattering in QCD. Cutkosky's rules in strong fields.

2 → N scattering in gravity in the Regge regime. Double copy structure of amplitudes

Scattering of gravitational shockwaves. Double copy structure of effective vertices and propagators

Classical double copy

RG evolution in GR. Radiation patterns, large N, the congruence of geodesics a la Raychaudhuri

The geometry of complexity and saturating the Bekenstein bound in QCD and gravity: The black hole N portrait

Double Copy: gluon → gravitational radiation in shockwave collisions

$$\begin{tikzcd}
\text{QCD at high occupancy} \arrow[d, "\text{Strong field semi-classical double copy}"'] & \text{perturbative QCD} \arrow[l] \arrow[d, "\text{BCJ double copy}"] \\
\text{Gravity at high occupancy} & \text{perturbative gravity} \arrow[l]
\end{tikzcd}$$

Bern, Carrasco, Johannson,
arXiv: 1004.0476

Monteiro,O’Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

$2 \to N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The S-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys.* 36 (1995) 6377; 3901 cites to date !**

In Acknowledgements:

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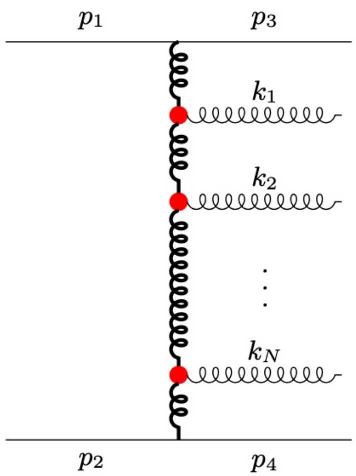
30+ years of work by ACV et al. exploring gravitational shockwave collisions in this 2-D EFT

Multiparticle production and gluon saturation in QCD:
from amplitudes to gluon shockwave collisions

BFKL: 2→ N QCD amplitudes in Regge asymptotics*



Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(``slow”) but hard in transverse momentum – weak coupling Regge regime

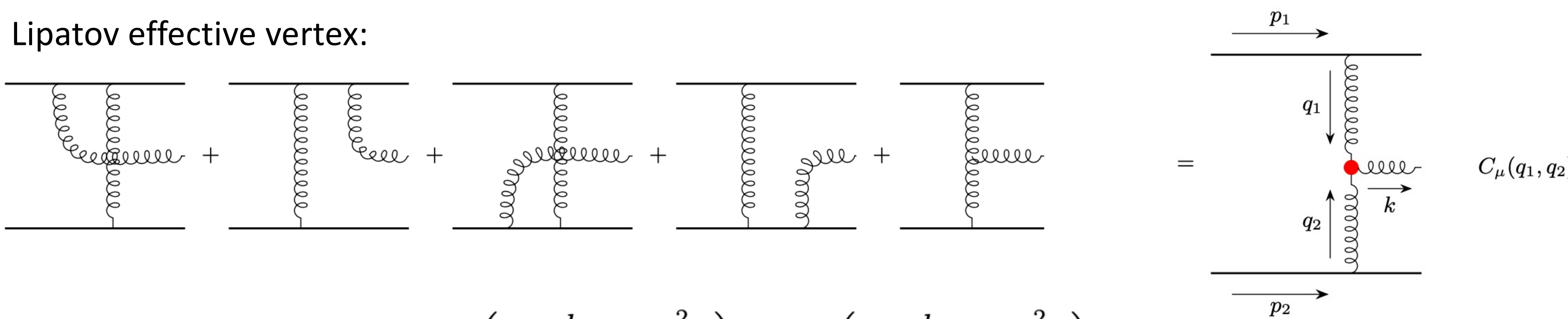
RG description rapidity of evolution given by the BFKL Hamiltonian
Very rapid growth of the amplitude with energy

A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0 + \alpha' \, |t|$ BFKL pomeron

* Asymptotics is the calculus of approximations. It is used to solve hard problems that cannot be solved exactly and to provide simpler forms of complicated results

BFKL: Building blocks

Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu\ C^\mu$ =0

BFKL: Building blocks

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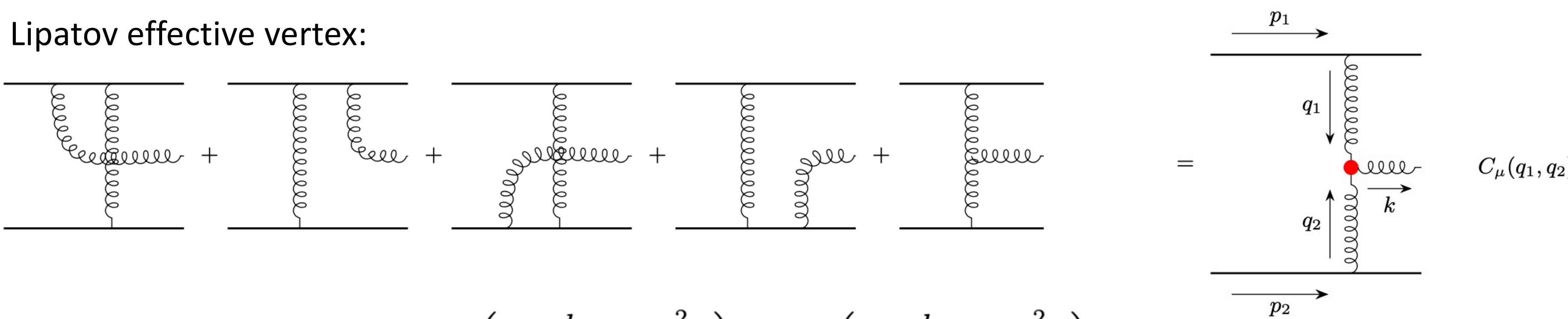
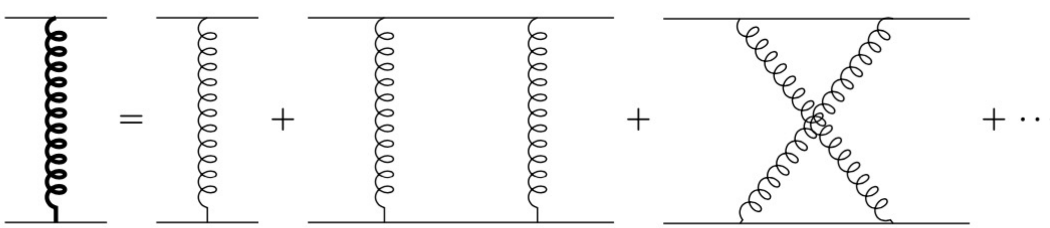


$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu\, C^\mu = 0$

Reggeized gluon:

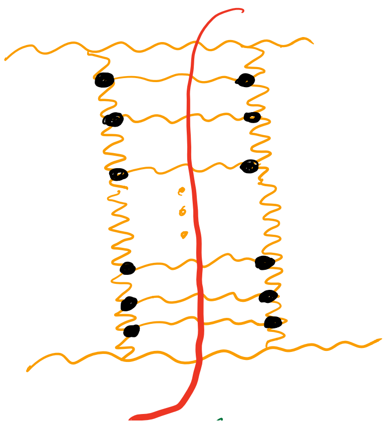


$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c\, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q} - \boldsymbol{k})^2}\,, \qquad t = -\boldsymbol{q}^2$$

$2 \rightarrow N + 2$ amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



$$\mathrm{Im}\,A(s,t) \propto \sum_{n=0}^{\infty} (\alpha_s C_T)^{n+2}$$

$$\times \int \prod_{i=1}^{n} \frac{dy_i}{4\pi} \prod_{j=1}^{n+1} \frac{d^2 q_{jl}}{(2\pi)^2}$$

$$\times\ 2is \prod_{l=1}^{n+1} \frac{1}{t_l t_{l'}} e^{(y_{l-1}-y_l)[\alpha(t_l)+\alpha(t_l')]}$$

$$\times \prod_{m=1}^{n} (C_\mu C^\mu)[q_m, q_{m+1}]$$

C_T is color factor

Phase space factors

Reggeized propagators on both sides of cut

Product of Lipatov vertices

$$\sigma_{tot} = 2\,\mathrm{Im}\,A(s,t=0)$$

$$= s^\lambda \text{ with } \lambda = \frac{4\alpha_s N_c \ln_e 2}{\pi}$$

$$\simeq 0.5 \text{ for } \alpha_s = 0.2$$

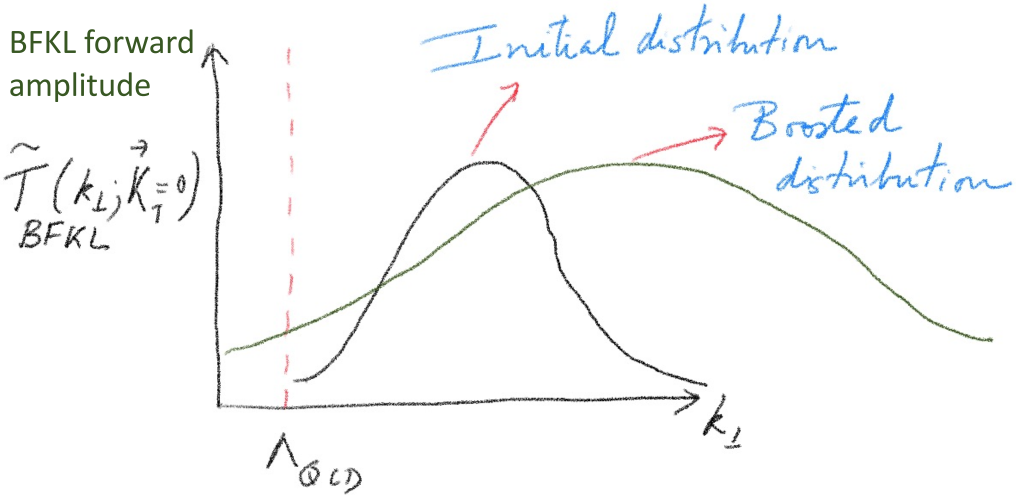
Real and virtual corrections combine to cancel infrared divergence !

Strongly violates Froissart bound

Resummed NLO BFKL : $\lambda \approx 0.3$

Excellent review of state-of-the art:
Del Duca, Dixon, arXiv:2203.13026

BFKL: infrared diffusion and gluon saturation



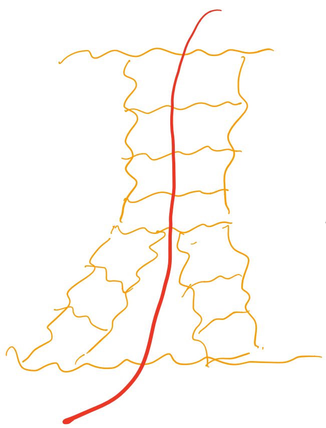
For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251

NLL BFKL does not cure infrared diffusion

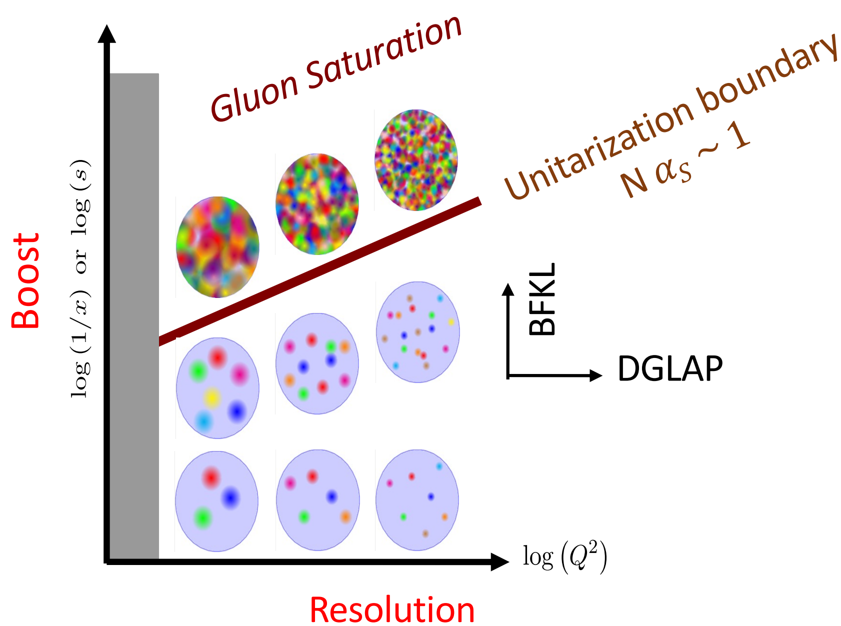
Gluon saturation cures infrared diffusion

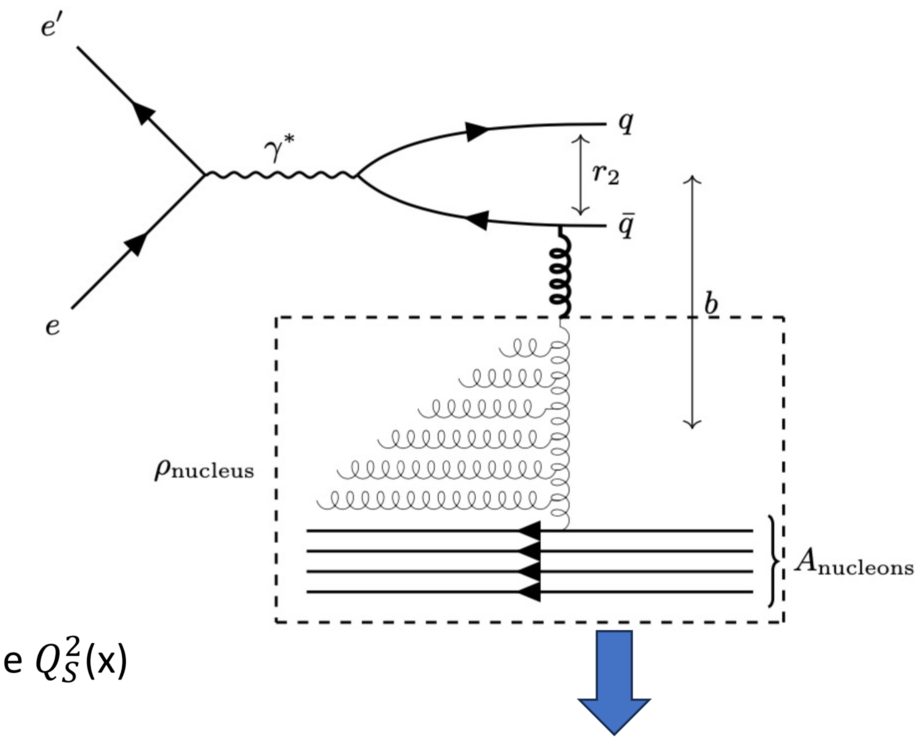


+ other higher twist cuts of O(1) when gluon occupancy $N \equiv \dfrac{xG_A(x, Q_S^2)}{2(N_c^2-1)\pi R_A^2 Q_S^2} = \dfrac{1}{\alpha_S(Q_S)}$

Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

Gluon saturation: classicalization and unitarization of cross-sections





Saturation – nontrivial fixed point – defines emergent scale Q_S^2(x)

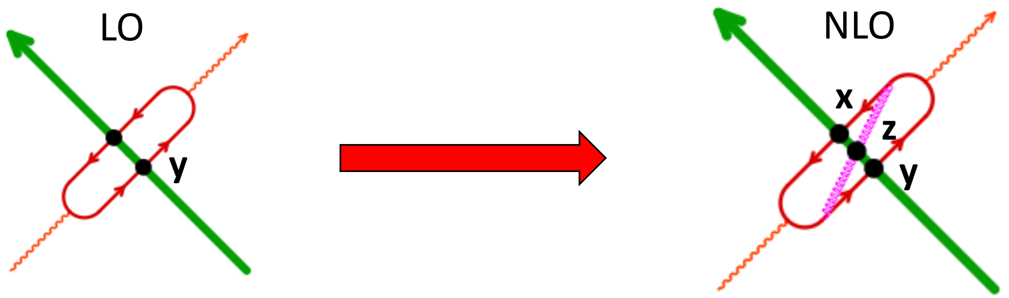
Color Glass Condensate EFT:
Powerful RG picture describes nonlinear (multi-Pomeron) evolution with rapidity – at NLLx accuracy for multiple final states

CGC review: Gelis,Iancu,Jalilian-Marian, RV:arXiv 1002.0333

Dense close-packed ($1/Q_S$) classical blob gluon "shockwave"

Reggeized gluon as field sourced blob's color charge density

DIS: dipole evolution in gluon shockwave background



Path ordered 2-D Wilson lines describe transition between two pure gauges on either side of shockwave

"B-JIMWLK" RG eqns. for Wilson-line correlators: Eg. 2-point "dipole"

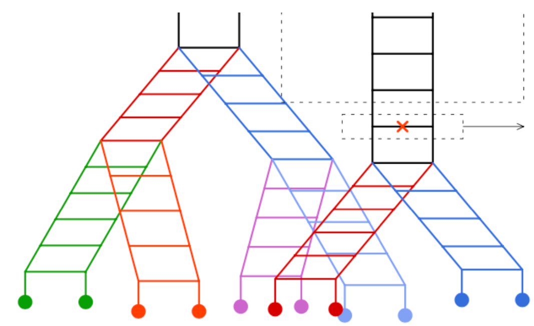
$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\,\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Y = Ln(1/x)

Closed form expression for A >>1, Nc→ ∞: non-linear Balitsky-Kovchegov (BK) eqn.

Evolved shockwave scattering off dipole probe contains all-twist multi-pomeron "fan" diagrams

BFKL obtained as the leading twist result…



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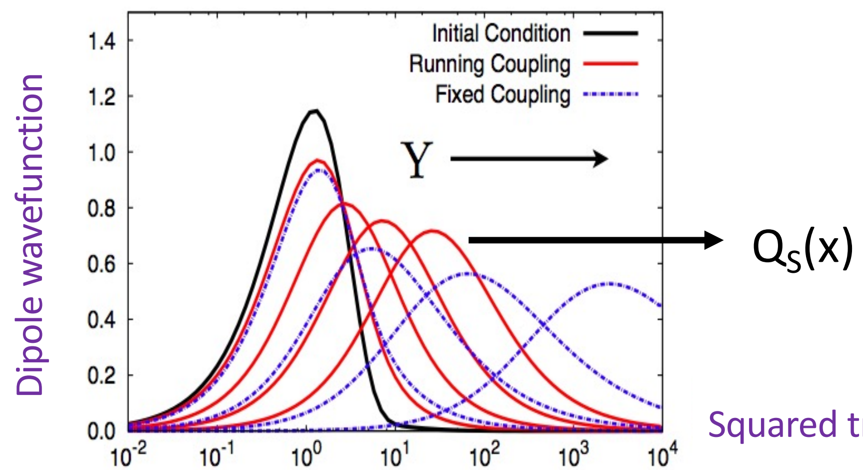
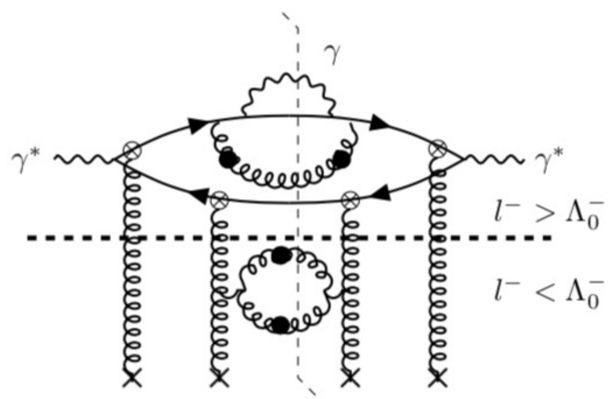


Squared transverse momentum



State-of-the-art:

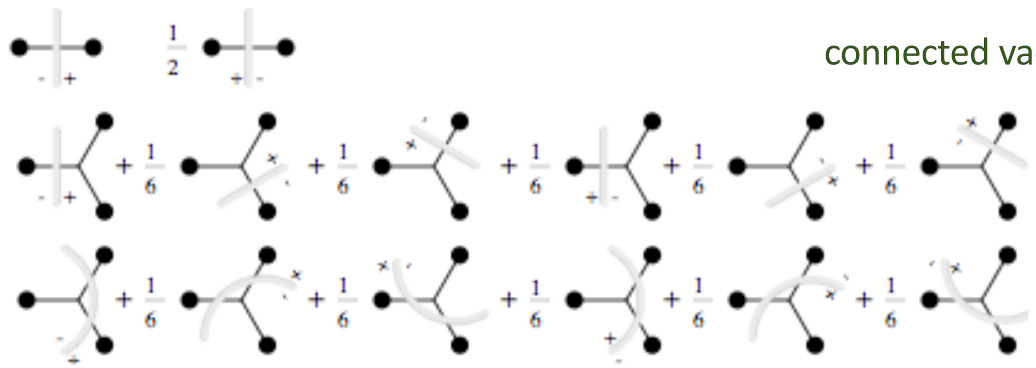
Significant amount of phenomenology describing small x HERA, RHIC, LHC data
- precision tests at EIC in future

"impact" factors for multiple final states to NLO accuracy

Small x RG evolution of shockwave to NLLx accuracy

General all-order formalism: Cutkosky's rules in strong fields

$$2\,\mathrm{Im}\sum_{\mathrm{conn.}} V =$$



connected vacuum graphs in $\lambda\phi^3$

Propagators on Schwinger-Keldysh contour

Well-known example: Schwinger pair production in strong field QED

Simple understanding of "AGK cutting rules" of Reggeon Field Theory:
combinatorics of cut and uncut sub-graphs contributing to a given multiplicity

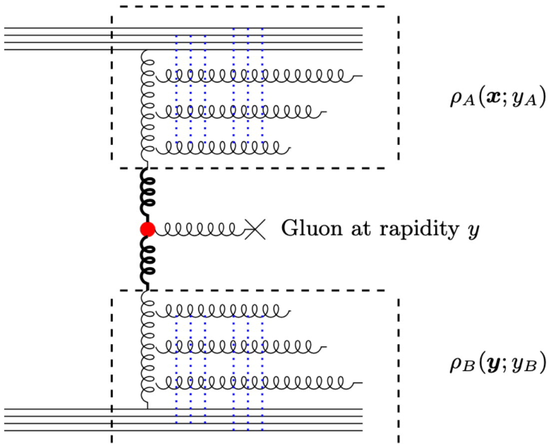
AGK: Abramovsy,Gribov,Kancheli

- Very general consequence of unitarity in strong fields
- Independent of language of Pomerons and Reggeons

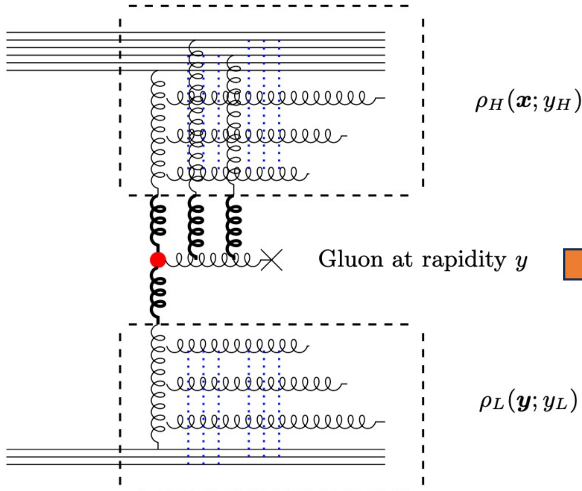
Paradigm shift? Perhaps Pomerons best viewed as simplest constructions enforcing strong field unitarity rather than fundamental objects

Gelis,RV: hep-ph/0601209, hep-ph/0608117

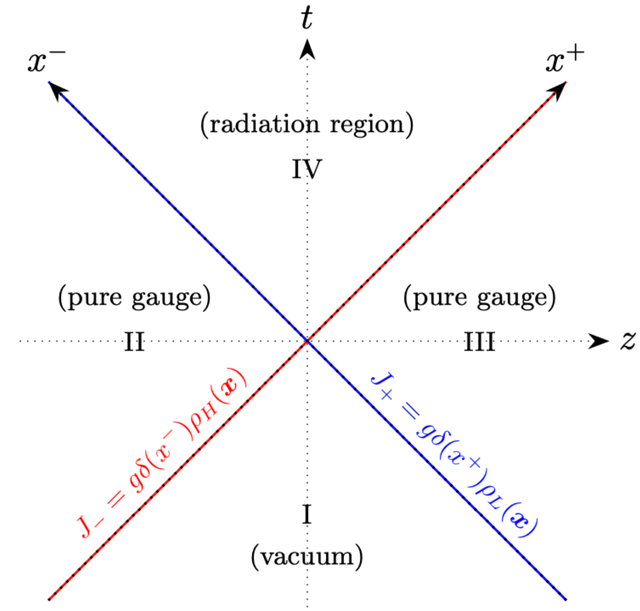
Gluon shockwave collisions: Lipatov vertex and reggeization



"dilute-dilute" collisions
$\left(\frac{\rho_A}{\Delta_\perp^2}, \frac{\rho_B}{\Delta_\perp^2} \ll 1\right)$



"dilute-dense" collisions
$\left(\frac{\rho_L}{\Delta_\perp^2} << 1, \frac{\rho_H}{\Delta_\perp^2} \sim O(1)\right)$



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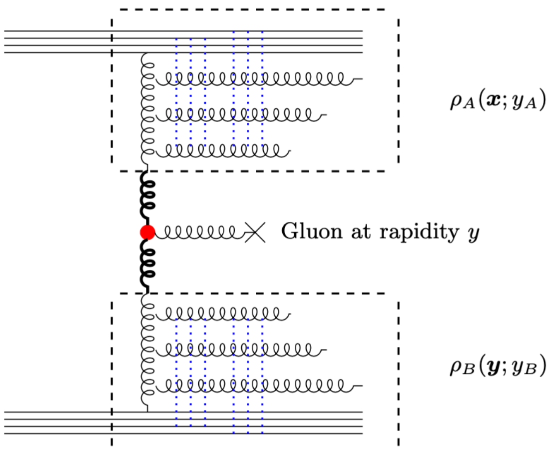
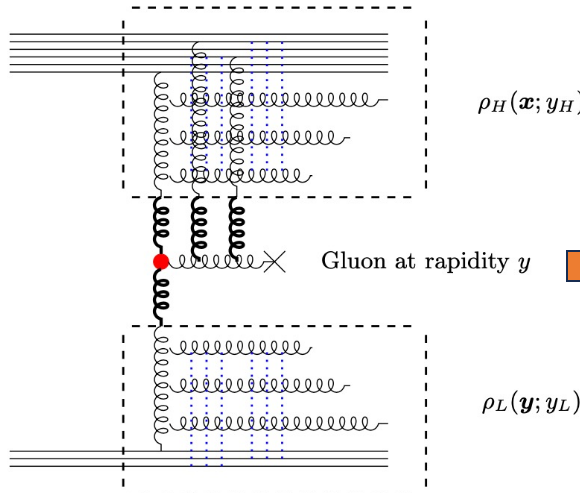
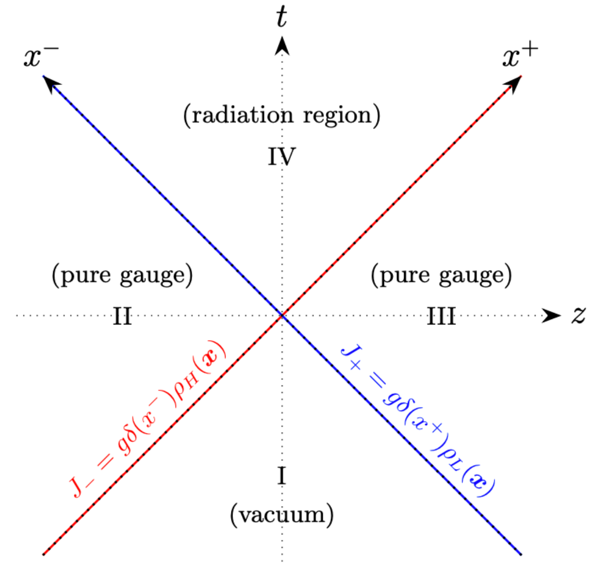






Weizsäcker-Williams gluon radiation field in light cone gauge

$$a_i(k) = -\frac{2ig}{k^2 + i\epsilon} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \left(q_{2i} - k_i \frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2} \right) \frac{\rho_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} \left(U(\boldsymbol{k} + \boldsymbol{q}_2) - (2\pi)^2 \delta^2(\boldsymbol{k} + \boldsymbol{q}_2) \right)$$

Lipatov vertex
in A^- = 0 gauge

reggeized gluons from
semi-classical source dists.

$$U(x^-, \boldsymbol{x})\delta\left(x^+\right) = \exp\left(ig \int_{-\infty}^{x^-} dz^- \bar{A}_-(z^-, \boldsymbol{x}) \cdot T \right)$$

$$\bar{A}_\mu(x^-, \boldsymbol{x}) = -g\delta_{\mu-}\delta\left(x^-\right) \frac{\rho_H\left(\boldsymbol{x}\right)}{\Box_\perp}$$

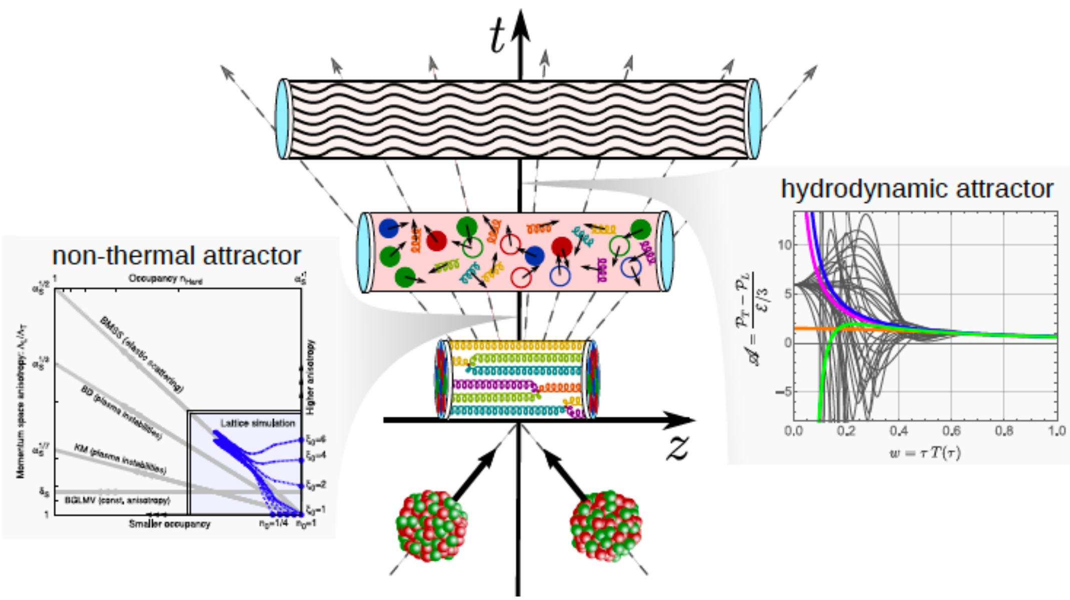
Ln (U) → *reggeized gluon*

Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

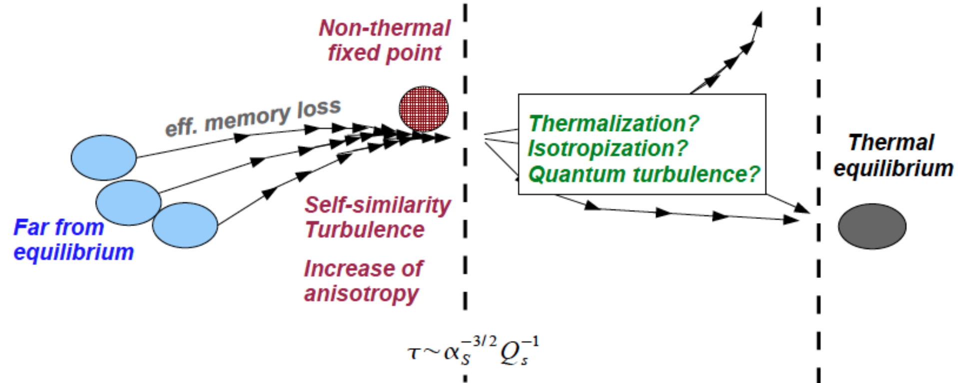
Jalilian-Marian,Jeon,RV (2000); Caron-Huot (2013)

Dense-dense shockwave collisions: heavy-ion collisions

Quark-Gluon Plasma undergoing
hydrodynamic expansion



Collision of overoccupied Color Glass Condensate shockwaves



Thermal soft gluon bath for $\tau > \dfrac{1}{\alpha_S^{5/2}} \dfrac{1}{Q_S}$

Thermalization temperature: $T_i = \alpha_S^{2/5} Q_S$

Very rapid thermalization
as $\alpha_S(Q_S) \rightarrow 0$ and $Q_S \rightarrow \infty$

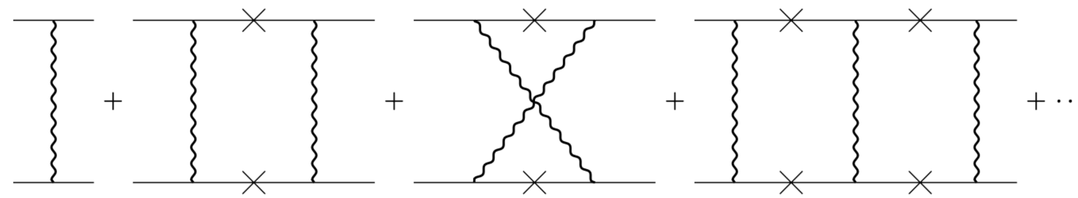
Baier, Mueller, Schiff, Son,
hep-ph/0009237

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

Multiparticle production and saturation in gravity: from amplitudes to shockwave collisions

From QCD to gravity in Regge asymptotics: reggeization

In Einstein gravity, at large impact parameters, the dominant contribution is eikonal scattering



$$i\mathcal{M}_{\rm Eik} = 2s \int d^2\boldsymbol{b}\; e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right) \qquad \text{with} \quad \chi(\boldsymbol{b},s) = \frac{\kappa^2 s}{2}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2}e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

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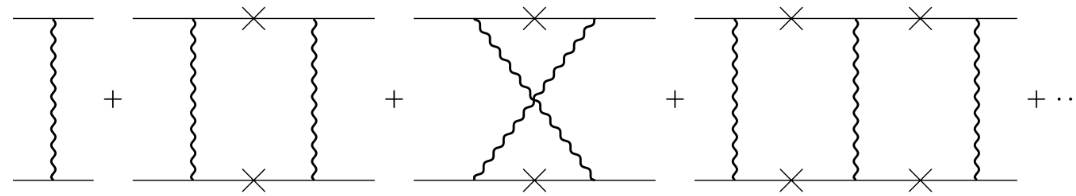

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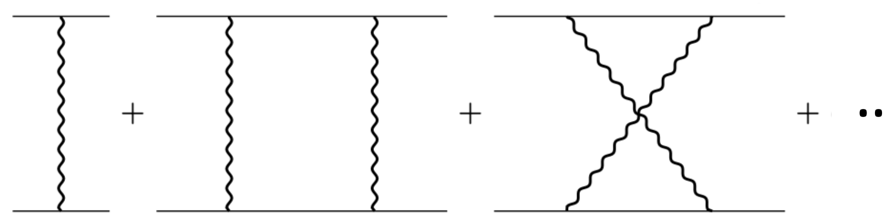
Genuine loop contributions formally suppressed by R_S^2/b^2

$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(\underbrace{-i\pi s \log\left(\frac{-t}{\Lambda^2}\right)}_{\text{Eikonal}} + \underbrace{t \log\left(\frac{s}{-t}\right)\log\left(\frac{-t}{\Lambda^2}\right)}_{\text{Loop}}\right)$$

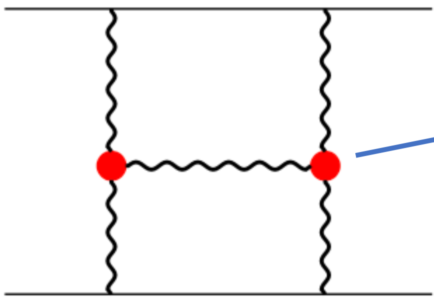


Graviton Regge trajectory: $\alpha(t) = -\kappa^2 t \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q}-\boldsymbol{k})^2} \left[(\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}))^2 \left(\frac{1}{\boldsymbol{k}^2} + \frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2} \right) - \boldsymbol{q}^2 \right] , \qquad \boldsymbol{q}^2 = -t$

The IR virtual divergence cancels in the inclusive cross-section

Lipatov, PLB 116B (1982); JETP 82 (1982)

From QCD to gravity in Regge asymptotics: Lipatov vertex



H-diagram of Amati, Ciafaloni, Veneziano

Gravitation Lipatov vertex:

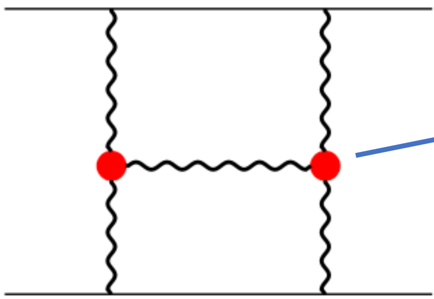
$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \frac{1}{2}\underbrace{C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\substack{\text{Double copy of}\\ \text{QCD Lipatov vertex}}} - \frac{1}{2}\underbrace{N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\substack{\text{Double copy of}\\ \text{QED Bremsstrahlung vertex}}}$$

$$N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}\right)$$

In amplitudes language, extra terms in double copy are imposed by so-called Steinmann relations
- required by unitarity to cancel spurious poles of energy variables ($s_1= (k+p_1)^2$ and $s_2=(k+p_2)^2$) in overlapping channels

Eg. Sabio Vera, Campillo, Vasquez-Mozo, 1112.4494

From QCD to gravity in Regge asymptotics: Lipatov vertex



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Gravitation Lipatov vertex:

$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \underbrace{\frac{1}{2}C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\text{Double copy of QCD Lipatov vertex}} - \underbrace{\frac{1}{2}N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\text{Double copy of QED Bremsstrahlung vertex}}$$

$$N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}\right)$$

S-matrix power counting a la ACV:

$$S = e^{2i(\delta_0+\delta_1+\delta_2+\cdots)}$$

$\delta_0 = Gs\log\left(\frac{L}{b}\right)$,

Leading Eikonal term (real)

$\delta_1 = \frac{6G^2 s}{\pi b^2}\log s$,

Sub-leading quantum gravity correction $\sim\frac{l_P^2}{b^2}$

$\delta_2 = \frac{2G^3s^2}{b^2}\left[1+\frac{i}{\pi}\log s\left(\log\frac{L^2}{b^2}+2\right)\right]$

Sub-leading loop contribution $\sim\frac{R_S^2}{b^2}$
- includes absorptive piece

$\boxed{\delta_2 \gg \delta_1 \text{ for } R_S \gg l_P}$

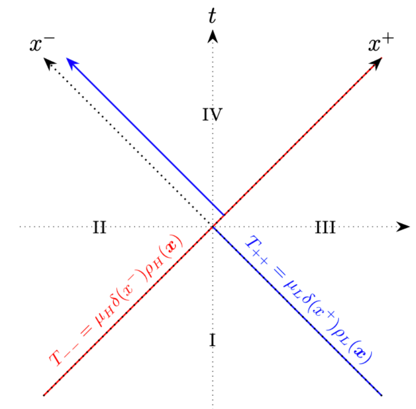
Shockwave collisions in general relativity

Aichelburg-Sexl shockwave metric of shockwave

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-,\boldsymbol{x})\left(dx^-\right)^2$$

with $f(x^-,\boldsymbol{x}) = 2\kappa^2\mu_H\delta(x^-)\dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi}\mu_H\delta(x^-)\displaystyle\int d^2\boldsymbol{y}\ \ln\Lambda|\boldsymbol{x}-\boldsymbol{y}|\rho_H(\boldsymbol{y})$

Soln of Einstein's eqns sourced by the EM tensor $T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H\delta(x^-)\rho_H(\boldsymbol{x})$



$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

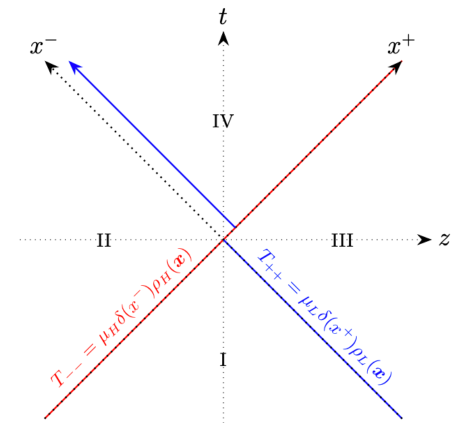
$\kappa^2 = 8\,\pi\,G$

Shockwave collisions in general relativity: single shock background

Aichelburg-Sexl shockwave metric of a shockwave

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\left(dx^-\right)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi}\mu_H \delta(x^-) \displaystyle\int d^2\boldsymbol{y}\ \ln \Lambda|\boldsymbol{x} - \boldsymbol{y}|\rho_H(\boldsymbol{y})$



$\mu_H = m_H\, \gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\, G$

Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

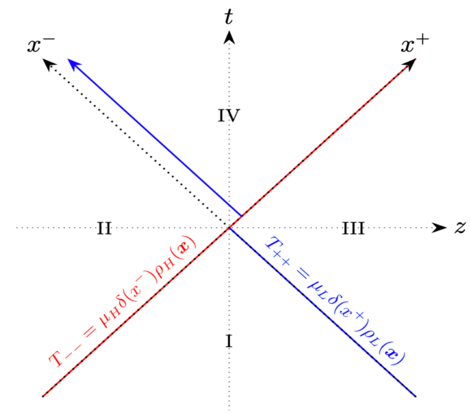
Fix light cone gauge $h_{\mu+}$=0. Find solution: $h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x}) h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\dfrac{1}{2}\displaystyle\int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\, \partial_+\right)$

Exactly analogous to the QCD case
with $A_- \to\ g_{--}$ and $T^a \to \partial_+$

Melville,Nachulich,Schnitzer,White,
arXiv:1306.6019

Shockwave collisions in general relativity: dilute-dilute approximation



$\mu_H = m_H \gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8 \pi G$

Now consider the interaction of the "dilute" source ρ_L with the dense ρ_H shockwave:

$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H \delta\left(x^-\right)\rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L \delta\left(x^+\right)\rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H \delta\left(x^-\right)\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

We decompose the perturbation $h_{\mu\nu}$ into a term linear in ρ_L and one bi-linear in $\rho_L \rho_H$

Linearized Einstein's equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2 \tilde{h}_{ij} - \Box \tilde{h}_{ij} = \kappa^2 \left[\left(2\partial_i\partial_j - \Box_\perp \delta_{ij}\right)\frac{1}{\partial_+^2}T_{++} + 2T_{ij} - \delta_{ij}T - \frac{2}{\partial_+}\left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right)\right]$$

$$\tilde{h}_{ij} \equiv h_{ij} - \tfrac{1}{2}\delta_{ij} h \text{ where } h = \delta_{ij} h_{ij}$$

Shockwave collisions in general relativity: geodesics

Unlike the QCD case, the sub-Eikonal contributions T_{+i}, T_{ij} are required for consistency of equations of motion

Since these are not uniquely fixed by energy-momentum conservation, the dynamics of the sources is needed to fix this. In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-g}} \int_{-\infty}^{\infty} d\lambda \; \dot{X}^\mu \dot{X}^\nu \; \delta^{(4)}(x - X(\lambda))$$

The solution of the corresponding null geodesic equations $\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \; , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

In the shockwave background is given by $X^- = \lambda \; , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

From the geodesic solutions, we can reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtains

Gravitational radiational field

$$\boxed{\tilde{h}_{ij}^{(2)}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}}$$

Gravitational Lipatov vertex

recovering Lipatov's result $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

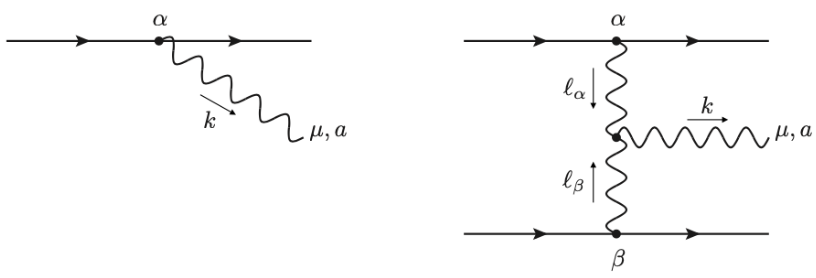
Compare to gauge theory radiation field

$$\boxed{a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}}$$

$$-if^{abc} T_b T_c C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \xleftrightarrow[\text{CK relation?}]{\text{Is there a}} s\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2)$$

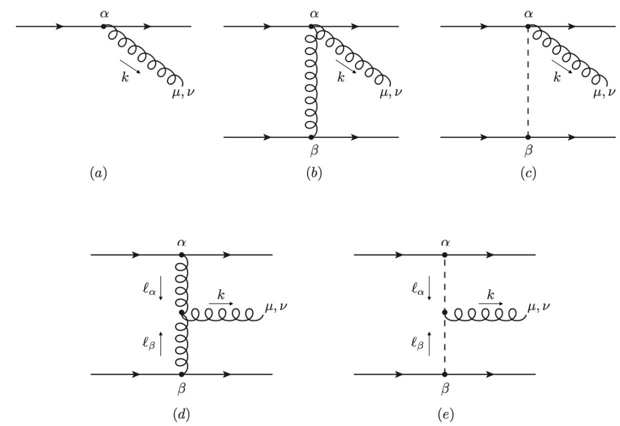
H.Johansson, A.Sabio Vera, E.Serna Campillo, and M.Vaszquez-Mozo, JHEP10,215(2013),arXiv:1307.3106 [hep-th]

Classical color-kinematic duality



(a) (b)

From Goldberger, Ridgway
arXiv:1611.03493



A color-kinematic duality does exist but it requires that one include sub-eikonal corrections to the Lipatov vertex

For this, require a detailed theory of sources: Yang-Mills+Wong equations for classical color sources c^a :

$$D_\mu F_a^{\mu\nu} = g J_a^\nu \qquad J_a^\mu(x) = \sum_{\alpha=1,2} \int d\tau\, c_\alpha^a(\tau) v_\alpha^\mu(\tau) \delta^d\left(x - x_\alpha(\tau)\right)$$

$$\frac{dc^a}{d\tau} = g f^{abc} v^\mu A_\mu^b(x(\tau)) c^c(\tau) \qquad \frac{dp^\mu}{d\tau} = g c^a F_{a\nu}^\mu v^\nu$$

Classical color-kinematic duality

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+c_1\cdot c_2\left\{\frac{q_1^2c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Classical color-kinematic replacement rule:

$$c_\alpha^a \to p_\alpha^\mu\ ,$$

$$if^{a_1a_2a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1,q_2,q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1-q_3)^{\nu_2}+\eta^{\nu_1\nu_2}(q_2-q_1)^{\nu_3}+\eta^{\nu_2\nu_3}(q_3-q_2)^{\nu_1}\right)$$

$$g \to \kappa\ ,$$

Gluon 3-pt vertex with f^{abc} stripped off

Classical color-kinematic duality

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\,\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\,\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu + \boldsymbol{q}_2^\mu + p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right) - p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+\, c_1\cdot c_2\left\{\frac{q_1^2 c_1^a}{p_1\cdot k}\left(-q_2^\mu + \frac{k\cdot q_2}{k\cdot p_1}p_1^\mu + \frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu - \frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right) + \frac{q_2^2 c_2^a}{p_2\cdot k}\left(-q_1^\mu + \frac{k\cdot q_1}{k\cdot p_2}p_2^\mu + \frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu - \frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $\qquad$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Performing the substitution, one finds the result we obtained by direct computation!

$$A^{\mu\nu}(k) = \frac{\kappa^3 s}{2\,k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\,\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\,\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\,\frac{1}{2}\,\left[C^\mu C^\nu - N^\mu N^\nu + \cancel{k^\mu\left(\frac{p_1^\nu}{p_1\cdot k}\boldsymbol{q}_1^2 + \frac{p_2^\nu}{p_2\cdot k}\boldsymbol{q}_2^2\right)}\right]$$

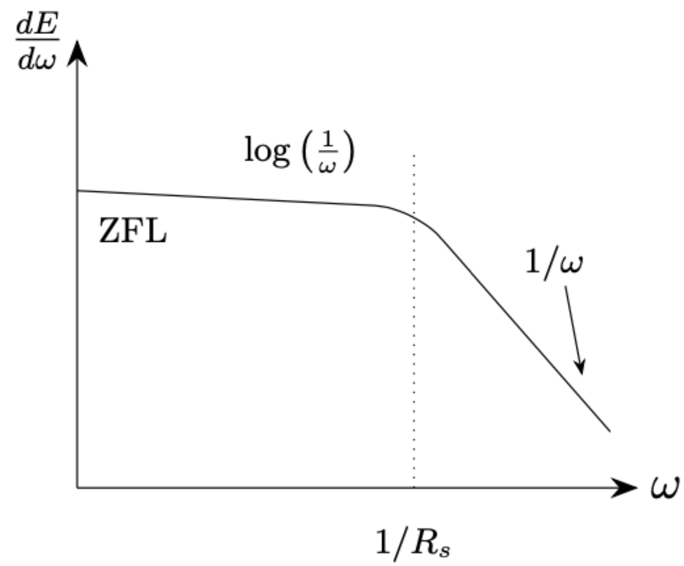
Unphysical – drops out when contracted with the gravitational polarization tensor

Next steps…

Compute WW spectrum in GR for dilute-dilute and dilute-dense cases

$$\frac{dE^{\mathrm{GW}}}{d\omega d\Omega} = \frac{1}{2\pi^2}\omega^2 \sum_\lambda \left| \mathscr{M}^{(\lambda)} \right|^2 \qquad \mathscr{M}^{(\lambda)} = k^2 \tilde{h}^{(2)}_{ij}(k)\varepsilon^{(\lambda)}_{ij}$$

Eg. Gruzinov, Veneziano, arXiv:1409.4555
Ciafaloni,Colferai,Coradeschi,Veneziano, arXiv:1512.00281

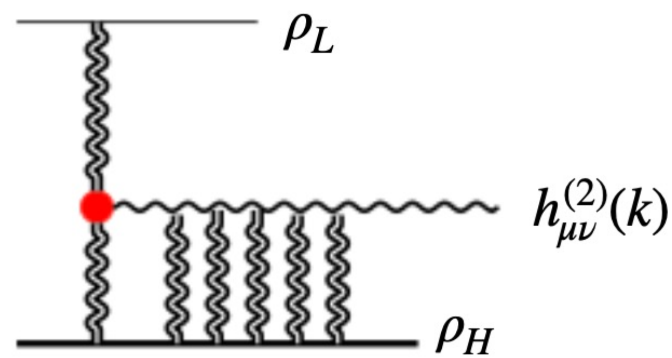


What is the shape of the spectrum as the compact objects get close. Solution of the Lipatov equation will tell us the spectrum as a function of impact parameter and rapidity

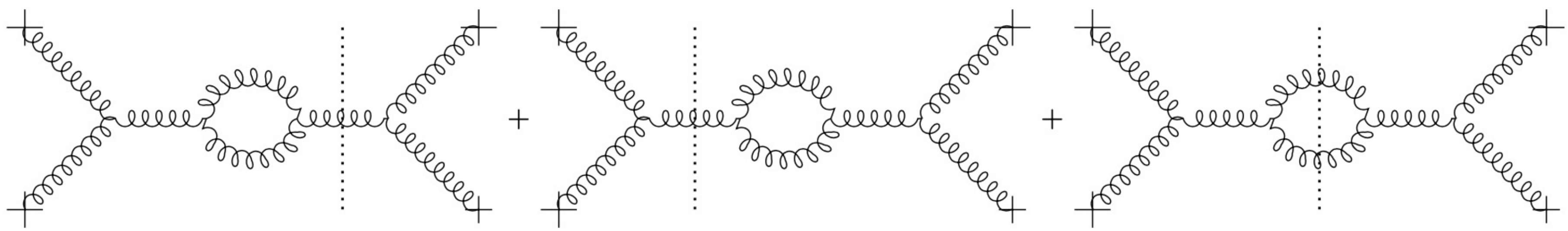
Amazingly, this has not been done…

Next steps…

Extend analysis to the dilute-dense
case to compute coherent multi-gravi-reggeon
contributions to the radiation spectrum



Next steps…



NLO corrections (absorptive piece of three loop diagram) in strong background field

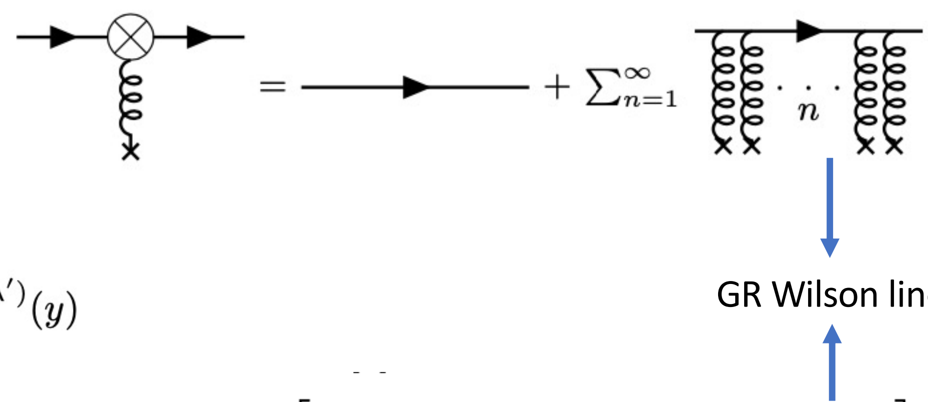
Rederive Lipatov's GR "BFKL equation" in shockwave language

Key ingredients are retarded shockwave propagators

Next steps...

Shockwave propagators
(graviton-reggeized graviton-graviton propagator)

Raj, RV, arXiv:2406.10483



$$G_{\mu\nu\rho\sigma}(x,y) = -\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon k^-} \sum_{\lambda\lambda'} h^{(\lambda\lambda')}_{\mu\nu,k}(x) h^{*(\lambda\lambda')}_{\rho\sigma,k}(y)$$

Soln. of small fluctuation equation: $h^{(\lambda\lambda')}_{\mu\nu,k}(x^-, x^+, \boldsymbol{x}) = e^{(\lambda\lambda')}_{\mu\nu}(k)\left[\Theta(-x^-)\, e^{-ikx} + \Theta(x^-)\, e^{-ikx} U_k(\boldsymbol{x})\right]$

$$\tilde{G}_{\mu\nu\rho\sigma}(p,p') = \tilde{G}^0_{\mu\nu\rho\sigma}(p)(2\pi)^4\delta^{(4)}(p-p') + \tilde{G}^0_{\mu\nu\alpha\beta}(p)\mathcal{T}^{\alpha\beta\gamma\delta}(p,p')\tilde{G}^0_{\gamma\delta\rho\sigma}(p')$$

where $\widetilde{G^0}$ is the free propagator and the shockwave effective vertex is

$$\mathcal{T}_{\mu\nu\rho\sigma}(p,p') = -\frac{1}{2}\left(\Lambda_{\mu\rho}\Lambda_{\nu\sigma} + \Lambda_{\mu\sigma}\Lambda_{\nu\rho} - \Lambda_{\mu\nu}\Lambda_{\rho\sigma}\right) 4\pi i (p')^- \delta(p^- - (p')^-) \int d^2\boldsymbol{z}\, e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{z}} \left(e^{if_1(\boldsymbol{z})p'_+} - 1\right)$$

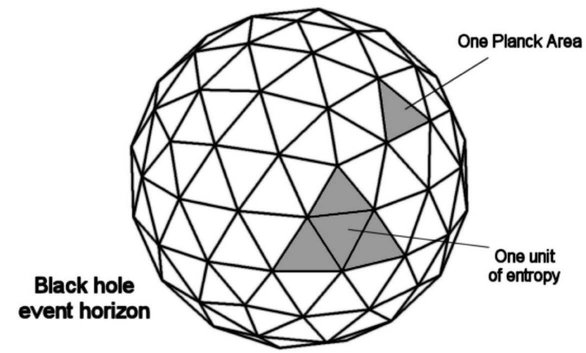
$$\Lambda_{\mu\nu} = \eta_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} \qquad f_1(\boldsymbol{x}) = \kappa^2 \mu \frac{\rho(\boldsymbol{x})}{\Box_\perp}$$

Remarkably, they satisfy double-copy relations to the QCD shock wave propagators

CGC-Black Hole correspondence

Similar deas developed previously by Gia Dvali and collaborators describe Black Holes as overoccuped graviton states

Dvali, RV conjecture: CGC-BH graviton states at max. occupancy $N = \frac{1}{\alpha}$



Micro-states of the CGC and BHs are Goldstone modes
of broken global symmetries.
These micro-states saturate the Bekenstein area law
for the maximal information for a state of given in a finite size region

$S_{Bekenstein} = f_G^2 * \text{Area}$

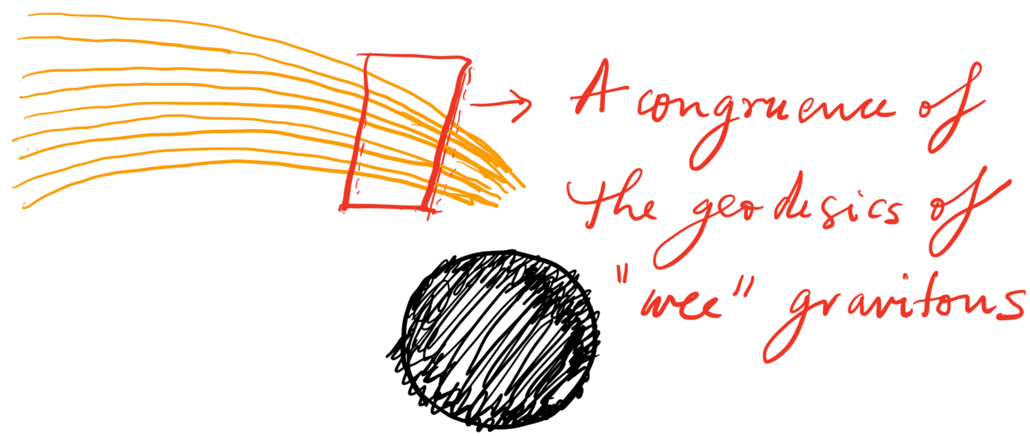
The Goldstone decay scale takes the universal form $f_G^2 = \frac{Q_S^2}{\alpha}$
In gravity it is the squared Planck mass. In QCD, it is a screening scale

Dvali, RV, PRD (2022)

As for the CGC case in the IR, can we understand BH formation as a non-trivial UV fixed point of RG evolution ?

Geodesic congruence: the geometry of quantum information

The Raychaudhuri equation
- key in Hawking-Penrose singularity theorems :



Volume change of geodesic convergence

$$\dot{\theta} = -\Omega^i{}_j \Omega^j{}_i + K^i{}_i$$
$$= -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} + K^i{}_i.$$

Bulk scalar | Shear tensor | Rotation tensor | Includes Ricci curvature + stochastic graviton noise

H.-T. Cho and B.-L Hu, arxiv:2301.06325
M. Parikh, F. Wilczek, G. Zaharaide, PRL (2021)

Remarkably, the Raychaudhuri equation can be rephrased as the Bishop-Gromov upper bound on the "complexity volume in D-1 dimensions" of gate complexity in quantum information
– realizing Nielsen's geometric picture (see Ananda Roy's talk)

A. R. Brown, arXiv:2112.05724

Pure speculation: Can this complexity picture provide further insight into the RG fixed point of BH formation?