Resummation of Cosmological Correlators

Matthias Nowinski

May 2025

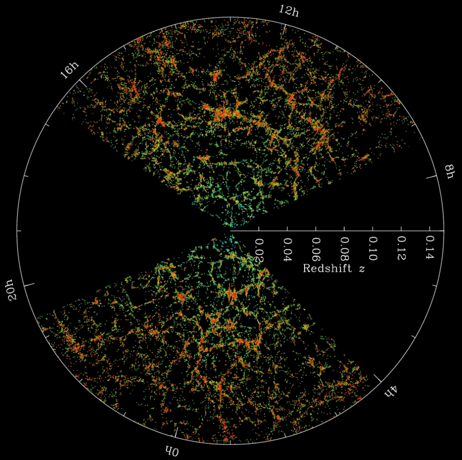
ORIGINS PhD Days

Table of Contents

Motivation

Galaxy Distribution

M. Slanton and Sloan Digital Sky Survey, SDSS map,
https://classic.sdss.org/includes/sideimages/sdss_pie2.php



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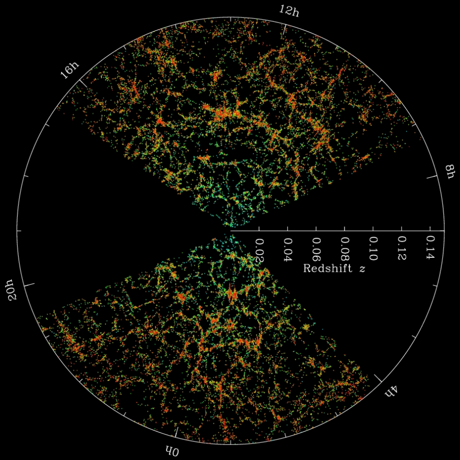

- Distribution of galaxies has Large Scale Structure (LSS)

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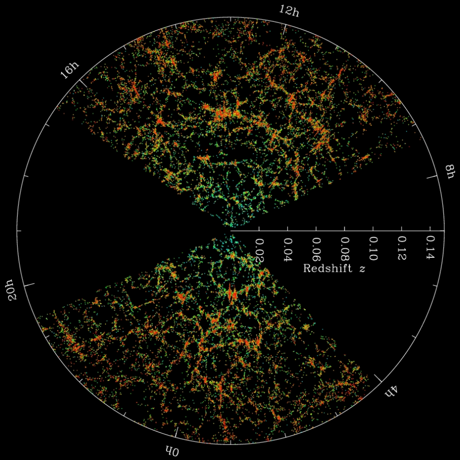
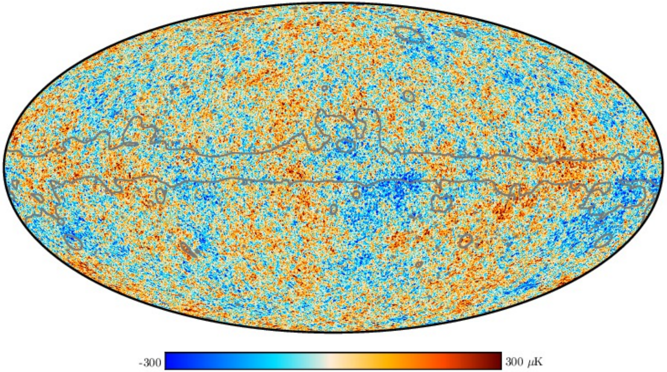


- Distribution of galaxies has Large Scale Structure (LSS)
- Where does this structure come from?

Cosmic Microwave Background

Aghanim, N. et al. *Astron. Astrophys.* **2020**, *641*, A1



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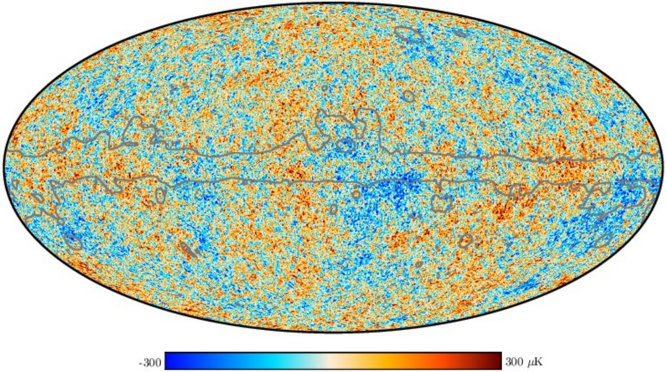

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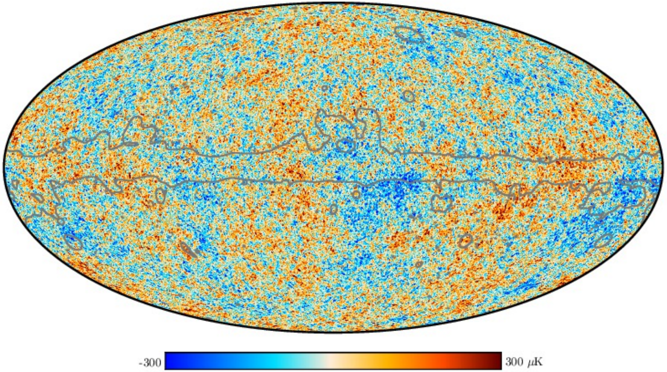
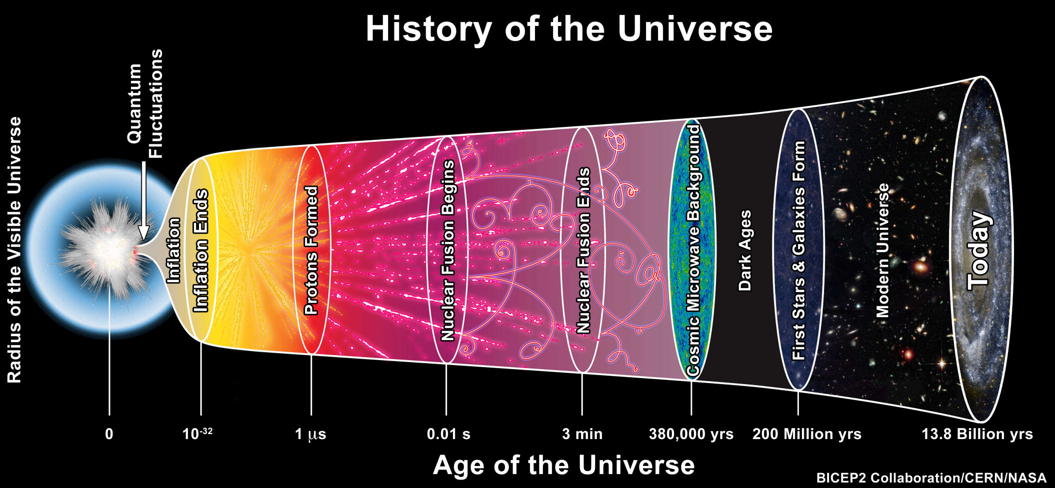


- Temperature anisotropies also have structure
- Common origin with LSS?

BICEP 2, CERN, NASA; History of the Universe,
https://de.m.wikipedia.org/wiki/Datei:The_History_of_the_Universe.jpg



Background

Inflation and Spacetime

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- Conformal symmetry: Scaling $z \to \lambda z$, $p \to p/\lambda$ (among others)

Cosmological Correlators

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- LSS and CMB structure can be traced to expectation values on future boundary

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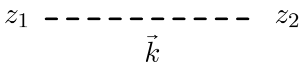
- Gives rise to 2 fields in EAdS

Feynman Rules

Feynman Rules

Chowdhury, C. et al. *JHEP* **2025**, *03*, 007



$$= \frac{(z_1 z_2 H^2)^{1-\epsilon}}{\pi C_\epsilon} \int_{-\infty}^{\infty} \mathrm{d}p \frac{\cos(pz_1)\cos(pz_2)}{(p^2 + \vec{k}^2)^{1+\epsilon}},$$

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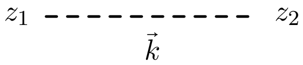

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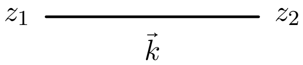

$$= -\frac{(z_1 z_2 H^2)^{1-\epsilon}}{\pi C_\epsilon} \int_{-\infty}^{\infty} \mathrm{d}p \frac{\sin(pz_1)\sin(pz_2)}{(p^2 + \vec{k}^2)^{1+\epsilon}},$$

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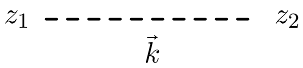

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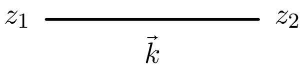

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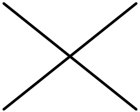
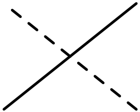
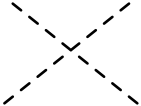


$$= \frac{\lambda}{4!} C_{2\epsilon},$$



$$= -\frac{6\lambda}{4!} C_{2\epsilon},$$



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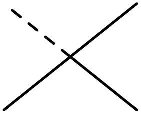
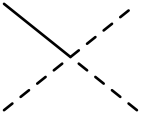


$$= -\frac{8\pi\epsilon\lambda}{4!},$$



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ϵ regulator

Loop Computations

L-Loop Diagrams

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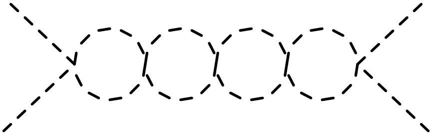

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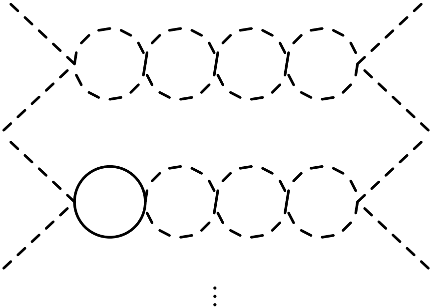

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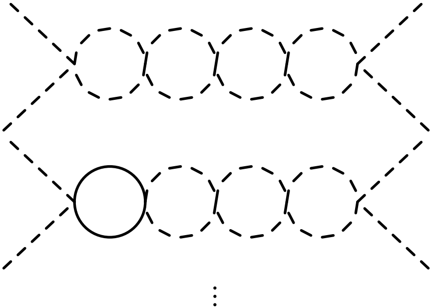

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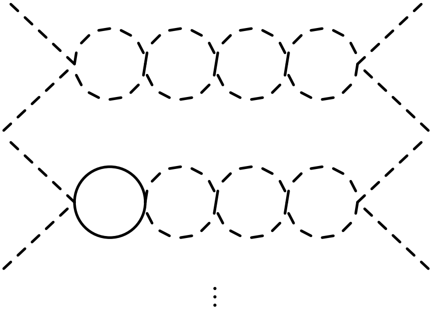

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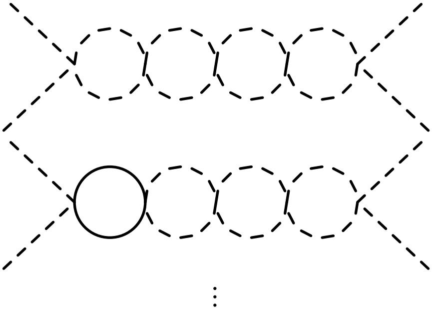

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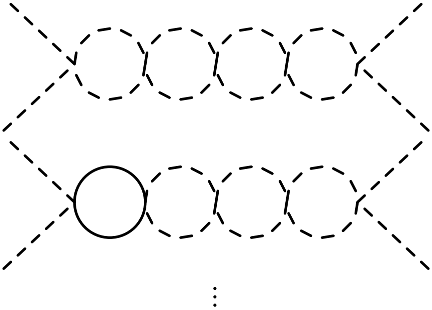

- Symmetry factors the same
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- Use flat space techniques for loop integral
- Renormalize with counter-terms

Loop Integrals

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- Sum of all terms contributing at L-loop:

$$I_L = \lambda^{L+1} K^L \int_0^\infty \frac{\mathrm{d}z_1}{z_1^2} \mathrm{d}z_2 \ldots \mathrm{d}z_L \frac{\mathrm{d}z_{L+1}}{z_{L+1}^2} \bar{G}_\epsilon^4(z_1, k_1, k_2; z_{L+1}, k_3, k_4)$$

$$\times \int_{-\infty}^\infty \mathrm{d}p_1 \ldots \mathrm{d}p_L \cos(p_1 z_1)\cos(p_1 z_2)\cos(p_2 z_2)\cos(p_2 z_3)\ldots\cos(p_L z_L)\cos(p_L z_{L+1})$$

$$\times \log\left(H^2 z_1 z_2 \frac{p_1^2 + k^2}{\mu^2}\right) \ldots \log\left(H^2 z_L z_{L+1} \frac{p_L^2 + k^2}{\mu^2}\right)$$

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- Omitting z_j^ϵ in regulator makes this easier: $\int_0^\infty \mathrm{d}z \cos(zp)\cos(zq) \propto \delta(p-q)$

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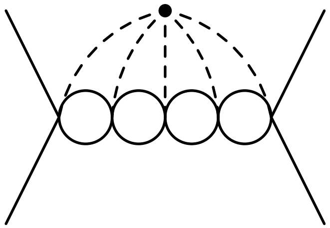
- Compare to new "cosmological dressing rules" formalism



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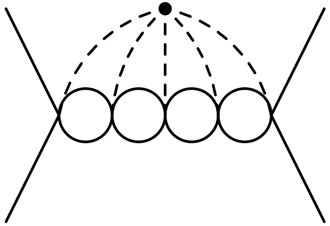

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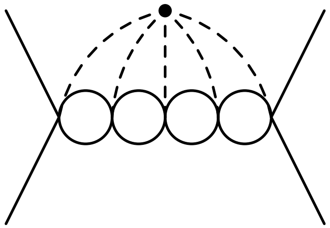

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Chowdhury, C. et al. **2025**

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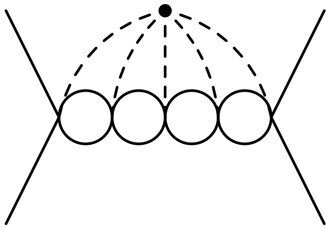

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- Is it possible to only regularize dotted lines connecting to external legs?
- With such a scheme, it would be possible to resum all L-loop terms