Introduction to Chiral Perturbation Theory

Exercises

Problem 1

Prove

$$\mathrm{Tr}\left(\partial_\mu U U^\dagger\right) = 0$$

for general $SU(N)$ case by considering an $SU(N)$-valued field

$$U(x) = \exp\left(i\,\frac{\phi^a \Lambda_a}{F_0}\right)$$

with $N^2 - 1$ traceless Hermitian matrices Λ_a and real fields ϕ^a.

Problem 2

Expand the mass term of $SU(3)$ ChPT Lagrangian

$$\mathcal{L}_m = \frac{F_0^2 B_0}{2}\,\mathrm{Tr}\left(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger\right),$$

where $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$, to second order in the physical fields and determine the squared masses of the Goldstone bosons.

Problem 3

Verify the transformation behavior

$$D_\mu A \to V_R (D_\mu A) V_L^\dagger,$$

where

$$D_\mu A = \partial_\mu A - i r_\mu A + i A l_\mu.$$

Problem 4

Consider the lowest-order πN Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(i\not{D} - m + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu)\Psi\,.$$

Assume that there are no external fields, so that

$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u\partial_\mu u^\dagger)\,, \quad u_\mu = i(u^\dagger \partial_\mu u - u\partial_\mu u^\dagger)$$

by expanding $u = \exp\left(i\,\phi/(2F)\right)$ in terms of pion fields derive the interaction Lagrangians containing one and two pion fields, respectively.

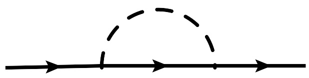

Figure 1: LO one-loop contributions to the nucleon self-energy.

Problem 5

Using pion-nucleon interaction obtained in Problem 4 obtain the pion-nucleon interaction vertex and calculate one-loop correction to the nucleon self-energy, shown in Figure 1.