Flavor Physics exercises

Methods of EFT and LFT

Exercise 1: Prove that any *quartet* combination

$$Q_{ijkl} = V_{ij} V_{kl} V_{il}^* V_{kj}^*, \text{ with } i \neq k, j \neq l,$$

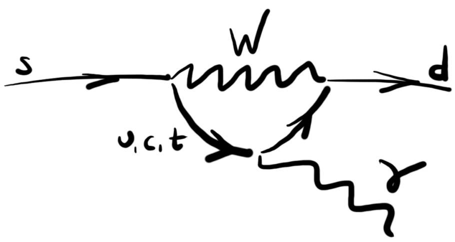
has the same imaginary part, with V the CKM matrix. Establish a relation between the quartets and the area and the angles of the unitarity triangle. As you can see, there are many Jarlskog invariants one can work with.

Exercise 2: Show explicitly how in a world with only two families of quarks the GIM mechanism cancels out the tree-level FCNC mediated by the Z^0 boson that the Cabibbo mixing introduces.

Exercise 3: Explain why in a world with two families there can't be any CP violation. There are several ways to reason this, get creative! (One can say that the realization of this fact gave Kobayashi and Maskawa a nobel prize).

Exercise 4: As a hint for the previous exercise, show that if two entries of the mass matrix M_u (**or** M_d) were equal, then we would be able to keep the CKM matrix real.

Exercise 5: Analyze the following diagram, corresponding to the $s \to d\gamma$ transition. Can we say that the t quark dominates the transition, as in the $b \to s\gamma$ case?



Exercise 6: Check that CPT invariance leads to $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

Exercise 7: Consider a mixed state of a particle X^0 and its antiparticle $\bar{X}^0$,

$$X_{L,H} = p \left| X^0 \right\rangle \pm q \left| \bar{X}^0 \right\rangle, \quad |q|^2 + |p|^2 = 1.$$

The Hamiltonian of the two-state system is given by

$$\mathcal{H} = \begin{pmatrix} M_{X^0} & M_C \\ M_C^* & M_{\bar{X}^0} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{X^0} & \Gamma_C \\ \Gamma_C^* & \Gamma_{\bar{X}^0} \end{pmatrix},$$

and the time evolution is governed by the Schödinger equation

$$i\partial_t \begin{pmatrix} \left| X^0(t) \right\rangle \\ \left| \bar{X}^0(t) \right\rangle \end{pmatrix} = \mathcal{H} \begin{pmatrix} \left| X^0(t) \right\rangle \\ \left| \bar{X}^0(t) \right\rangle \end{pmatrix}.$$

Assuming CPT invariance, calculate the mass eigenstates of the system, describe their time evolution, and from them write the time evolution of the interaction eigenstates (X^0 and $\bar{X}^0$).