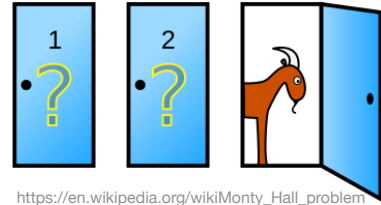


2. The Monte Hall problem

You take part in a gameshow. At one point in the show the host presents you with three doors, each hiding one prize. You get to choose one of the doors and get to keep whatever is behind it. Two of the doors are hiding a goat and one is hiding a sportscar. After you have made your choice the host, who knows which door is hiding the car, opens one of the doors you have not chosen, making sure he is revealing a goat. Now he asks you if you want to stick to your original choice or if you would like to get what is behind the third door. Should you change to the second door (assuming you prefer cars over goats)? Give a formal proof of your answer using Bayes' Theorem.

Hint: Assume the host to play fair and to always reveal a goat behind another door before one chooses.



3. Poisson probability distribution

Another discrete probability distribution that we'll be spending some time with in the class is the Poisson probability distribution:

$$P(k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k \in \{0,1,2,\dots,\infty\}$$

The following questions give you an opportunity to practice your discrete probability gymnastics, and prove some key properties of this distribution(!)

3a) Prove that the distribution is normalized

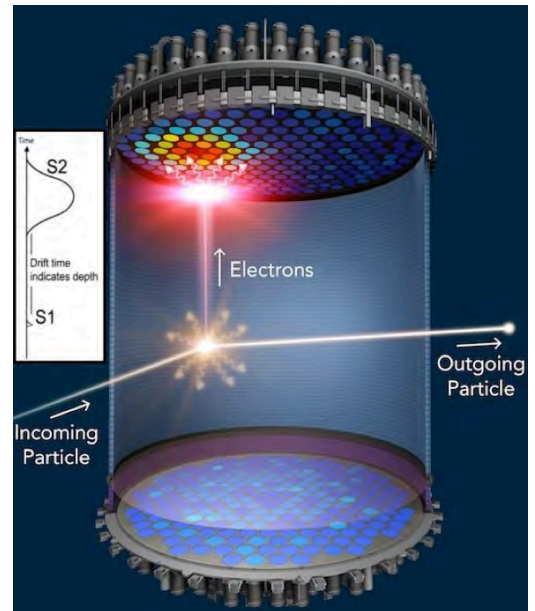
3b) Prove that the mean is λ

3c) Prove that the variance is λ

3d) Imagine you're a pheno PhD student building dark matter models. You just worked through an arduous calculation for your new model, and arrive at a prediction: you expect $\lambda = 1.5$ dark matter events per year to be observed by the currently running Lux Zeplin (LZ) experiment.

They're already taking data... so you have a prediction to compare against! With some trepidation (and gratitude to you your past self for enrolling in the ODSL Stats block course!)... you set out to calculate some probabilities.

If your model is true, what is the probability that LZ sees 6 or more events in a year?



In the LZ experiment, a dark matter particle recoils off a Xenon nucleus and give a light (S1) and charge (S2) signal recorded by photomultiplier tubes at the top of the detector. <https://lz.slac.stanford.edu/our-research/lux-zeplin-research>

If your model is true, what's the prob LZ didn't see any events in a year?